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Analysis of a proposal to implement the readiness-based sparing process in the Brazilian Navy

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THESIS

ANALYSIS OF A PROPOSAL TO IMPLEMENT THE READINESS-BASED SPARING PROCESS IN THE BRAZILIAN NAVY

by

Régis M. Nogueira

June 2017

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With the Brazilian Navy facing increasingly tight budget constraints as a result of the country's severe economic crisis, maintaining appropriate inventory levels of stochastic spare parts, to support complex weapon systems, has become more challenging. Currently, only the traditional item approach is utilized to determine inventory levels.

This research investigates whether implementing the U.S. DOD readiness-based sparing (RBS) methodology could provide the Brazilian Navy with greater cost savings, in comparison to the traditional item approach, without compromising the availability of supported weapon systems.

A case study is developed based on a helicopter engine and a selected group of its critical reparable components. The existing inventory of reparable components is evaluated against a range of efficient solutions computed using greedy heuristics.

The results indicate that the analyzed inventory could be dramatically reduced, while the expected average system availability would not be adversely affected. Moreover, potential cost savings could be leveraged if the proposed approach is applied to establish initial provisioning.

The implementation of readiness-based sparing within the Brazilian Navy is recommended, but a thorough investigation is proposed prior to a comprehensive implementation of the model. A phased approach is suggested by applying the methodology first for determining reparable spares initial provisioning.

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LIST OF ACRONYMS AND ABBREVIATIONS

AAM	aircraft availability model
ASM	aircraft sustainability model
Ao	operational availability
BN	Brazilian Navy
BNICP	Brazilian Navy inventory control point
BNSS	Brazilian Navy supply system
BO	backorders
COTS	commercial-off-the-shelf
DOD	Department of Defense
DI	due in
EBO	expected backorders
EOQ	economic order quantity
ICP	inventory control point
FMC	full mission capable
LORA	level of repair analysis
LRU	line replaceable unit
MC	mission capable
METRIC	Multi Echelon Technique for Recoverable Item Control
MIME	multi-indenture multi-echelon
MTBF	mean time between failures
MTTR	mean time to repair
OH	stock on hand
RBS	readiness-based sparing
SKU	stocking keeping unit
SINGRA	Sistema de Informações Gerenciais de Abastecimento
TAT	turnaround time
USAF	United States Air Force
USD(AT&L)	Under Secretary of Defense for Acquisition, Technology, and Logistics
USN	United States Navy

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Foremost, I am deeply grateful to Jesus Christ, the source of all wisdom, knowledge, and truth, for giving me strength, peace, and perseverance in the most difficult moments. He will never give you more than you can handle. His will is always good, pleasing, and perfect.

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I. INTRODUCTION

A. BACKGROUND

The task of managing spare parts of complex weapon systems is quite challenging. In general, spare parts are costly, with higher demand unpredictability and greater resupply lead times when compared with other categories of inventory items. Furthermore, the Brazilian Navy (BN) procures the majority of its weapon systems from foreign countries. Hence, there is a great dependence on international suppliers to provide spare parts, resulting in extended lead times and increased transportation costs, which intensifies the challenges faced by managers.

Moreover, Brazil has been facing an extremely severe recession. Brazil's economy contracted 3.8% in 2015, and 3.6 % in 2016, the worst recession in the last decades (Focus Economics, 2017). The economic crisis in Brazil is expected to persist in the next years. This context leads the Brazilian Navy (BN) to examine scenarios of increasingly tight federal budget constraints. Consequently, financial resources destined to the resupply of spare parts have been continuously shrinking, which dramatically affects weapon systems readiness levels.

The main purpose of the BN Maintenance System, as stated in the BN Directive EMA-420, is to keep the material ready for utilization, at the appropriate place, at the right time, according to its designed features and in the most economical manner, respecting the materials requirements (Estado-Maior da Armada [EMA], 2002). Material is broadly defined, encompassing items, equipment, sub-systems, and weapon systems. Clearly, readiness is a key objective, though cost-effectiveness is also an essential goal. Thus, just like other typical military structures, these parameters are major drivers for decisions on inventory needs.

Given that the maintenance environment is stochastic by nature, to increase material readiness spare parts should be appropriately kept in stock in order to be used quickly, once a failure occurs. The objective is to reduce the supported system downtime maintenance, increasing system availability. While a system's downtime costs in the

military environment are very difficult to monetize, they are incontestably high. In fact, it should be emphasized that a weapon system is valuable only if it is available to be used once required, otherwise, there is no utility for owning a weapon system (Jones, 2006, p. 10.1).

Demand forecast methods cannot precisely predict the range and the depth of the necessary spare parts to support the system. In short, “there is no magic formula that can be used to identify requirements for spare parts” (Jones, 2006, p. 18.1).

To address some of those challenges, several management actions have been applied in the BN to improve the management of the maintenance supply system. One example is the recent implementation of total asset visibility. Stock levels of organizational units were integrated to the BN Supply Management System, at the Inventory Control Point (ICP), with the goal to provide integrated Supply Chain information between retail and wholesale echelons. The total asset visibility allowed the execution of lateral transshipments between retail (or organizational) levels, resulting in improved fill rate measures, as well as mitigating the problem of excess in stocks.

Nonetheless, since the BN Supply Management Information System was implemented in 2001, the core of the spare parts determination process remains fairly unchanged. The traditional item-approach method is utilized to forecast spare parts inventory levels. This method consists of forecasting demand for each part individually, without considering the other parts and without measuring the expected effect on the supported system. A major drawback of this approach is that system availability and total spare parts investment are uncontrolled results of the model (Sherbrooke, 2004, p. 3).

Even though the current policy has been able to support systems under the circumstances described in this section, it is imperative to investigate other approaches, seeking better inventory management models to achieve greater efficiency.

In contrast with the traditional approach, Readiness-Based Sparing (RBS), as defined in the *DOD Manual 4140.01*, volume 2, is "a requirement determination process that computes the levels of secondary item spares needed to support a weapon system readiness goal at least cost" (USD(AT&L), 2014, p. 54). The same volume of the manual states the terms "readiness based requirements" and "sparing-to-availability" are synonymous with RBS.

To determine the spare parts requirements, RBS adopts a system-approach optimization technique, as opposed to the item-approach model, to compute the range and the depth of spare parts inventory levels, associating these spare parts investments costs to the availability of the supported systems. In this context, RBS will be investigated as an alternative approach to address the gaps of the current model adopted by the BN.

In the past decades, RBS has been successfully applied in the U.S. military services to optimize inventories, achieving higher readiness rates and cost savings (Office of the Assistant Deputy Under Secretary of Defense for Supply Chain Integration (OADUSD[SCI]), 2008, p. 19). This thesis will examine the implications of RBS to the BN.

B. RESEARCH QUESTIONS

The issues described in the previous section will be addressed by the following research questions:

1. Primary research question

Could RBS model provide improvement in readiness and/or cost savings if adopted by the BN?

2. Secondary research questions

- What are the underlying principles, concepts and applications of the RBS model?
- What are the major differences between the RBS and the model adopted by the BN?
- What are some advantages and limitations of applying the RBS model in the BN?

C. SCOPE

Spare parts can be categorized as reparable and consumable (non-reparable) items. Even though both types of items will be addressed in this research, the thesis will focus on reparable parts, characterized by high cost and low demand, as they are central to the development of system-approach models, due some particular characteristics, which are explained in the next chapter. Furthermore, while maintenance tasks can be distinguished as preventive and corrective, only sparing requirements due to corrective maintenance tasks resulting from stochastic failures are part of the study. Scheduled preventive maintenance is not analyzed.

D. METHODOLOGY

The thesis uses both qualitative and quantitative analytical tools to investigate the RBS methodology, and to compare it to the current model adopted by the BN. The qualitative analysis relies in literature provided by books, scholarly articles, technical reports, military directives, instructions and manuals. The quantitative approach consists of a case study of one BN weapon system and a selected group of its critical components.

The case study includes an optimization technique based on the RBS approach, using MS Excel to perform quantitative analysis, in order to evaluate alternatives of spare parts levels and the correspondent results in terms of the selected weapon system's availability, and total inventory investment cost. The same tool is utilized to generate the optimal availability versus total investment cost curve. Data from the current existing spare levels concerning a selected group of components will be the input in the model to compare the results with the RBS approach.

Part of the data employed in the quantitative analysis was provided by the BN Supply Chain Management System. Furthermore, due to the fact that key data elements required by the RBS computations are not captured by this system, complementary data were acquired directly by organizational-level unit.

E. STRUCTURE OF THESIS

This thesis is organized into five chapters. Chapter I introduced the research problem and motivation, presented the research questions, briefly described the scope and the methodology. Chapter II presents the literature review on the RBS system-approach optimization methods, covering their underlying principles, assumptions and general mathematical concepts. Chapter III describes a case study of a BN weapon system and selected group of its critical reparable components, based on the RBS approach. Chapter IV performs the quantitative analysis of the case study described in the previous chapter and presents the results. Chapter V presents the conclusions, recommendations, and suggestions for further research.

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II. LITERATURE REVIEW

A. INTRODUCTION

This chapter investigates the literature in system approach inventory models, also known as multi-item approach, based on the multi echelon technique for recoverable item control (METRIC). The chapter also covers underlying principles, assumptions, and general mathematical concepts, common to those system approach models.

B. SYSTEM APPROACH MODELS

The literature in system approach sparing models is rich. Several studies were developed over the last 50 years. While there is large number of system approach models, the review is restricted to METRIC model and its variations, given that these models represent the origin of the RBS methodology and have been extensively used by the military services to manage spare parts.

1. METRIC

A pioneer work towards the system approach methodology to the management of spare parts inventories was conducted by Sherbrooke (1968). The experiment, supported by RAND Corporation, was carried out in the United States Air Force (USAF), in a two-echelon system, consisting of one depot and two bases. The analysis was restricted to the management of aircraft recoverable parts stock levels. Sherbrooke explained that these items, characterized by high cost and low demand, were very representative in the total investment of spare parts in the USAF, and accounted "for about 52 per cent of the total investment in spare, or approximately 5 billion dollars" (Sherbrooke, 1968, p. 122). He added that maintenances tasks occurred either at the base or at the depot, depending on the complexity of repair.

The research presented the well-known mathematical model called METRIC, an acronym meaning "multi echelon technique for recoverable item control" which is claimed to be the first multi-item and multi-echelon spare parts optimization model (Sherbrooke, 1968, p.123).

To develop METRIC optimization model, Sherbrooke (1968) utilized the following general mathematical assumptions:

- Demand for items follows a Compound Poisson Process.
- Demand is stationary.
- Repair location depends on the complexity of the repair.
- Lateral resupply between bases is ignored.
- System is conservative, meaning there are no disposals (i.e., any defective item can be repaired to its serviceable condition.
- Items are not batched, which means that a repair or resupply is triggered whenever a demand occurs.
- All items have equal essentiality.
- Demand data from different bases can be pooled. (Sherbrooke, 1968, pp. 126–131)

While the assumptions above will be described throughout the development of this thesis, at this point, it is important to highlight the assumption that a*ll items have equal essentiality*. This assumption means that the unavailability of any recoverable part puts an aircraft out of service. Sherbrooke acknowledged that the criticality of recoverable items and bases differ; however, he considered reasonable to adopt *equal essentiality* assumption for all items as a good approximation (1968, p. 130).

Before describing the METRIC process, it is important to present two major supply chain performance measures. The first, and most widely utilized, is fill rate, defined as the number of requisitions immediately satisfied by on hand inventory divided by the total number of requisitions in the period. The second is backorder, defined as the number of orders that have not been filled yet.

In the METRIC model, a backorder occurs whenever there exists an unsatisfied demand at the base level. Sherbrooke selected backorders as the target performance measure on which to perform the optimization problem, because among other factors, the backorder measure takes into account the period length, whereas the fill rate measure

does not. The objective function of METRIC is to minimize the sum of expected backordered reparable items across all bases for a specific supported aircraft (p. 126).

The sum of expected backorders for various alternatives of stock levels, and the correspondent investment costs, provides a curve of optimal stock levels, such as the one in Figure 1.

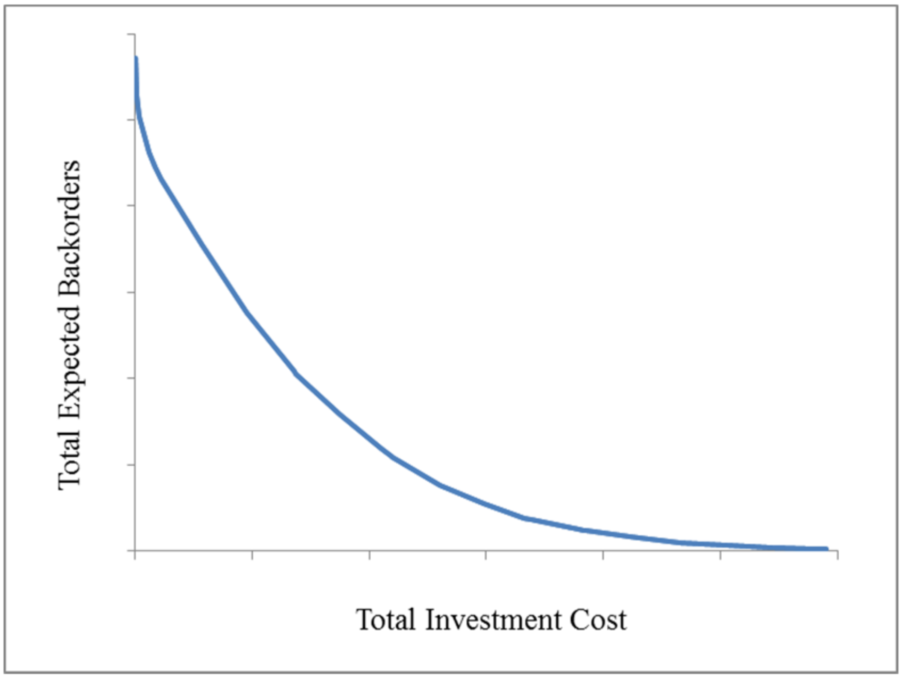


Figure 1. Total Expected Backorders versus Total Investment Cost Curve

The minimization of total expected backorders represents the maximization of system availability (Sherbrooke, 2004). Therefore, it is demonstrated that the optimal expected backorder curve can be converted instantaneously to an expected availability optimal curve, based on the total investment cost (p. 20). Figure 2 illustrates a typical optimal curve between system availability and investment cost.

A major advantage of the system approach, when compared to the item approach, is this ability to display an availability-cost curve of efficient sparing plans, for a group of recoverable items of a specific weapon system. In fact, Sherbrooke pointed out that the two approaches are interconnected, given that the efficient system approach solutions are

computed using the item approach outputs for a given set of parameters (Sherbrooke, 2004, p. 4).

To put it differently, the *system approach* is able to provide an *efficient frontier curve*, allowing managers to make decisions of spare parts investments based on the effect on the supported system's availability within budget constraints. Optimal solutions can be chosen from any point along the curve shown in Figure 2. Any point below the curve represents inefficiency, while points above the curve are not feasible.

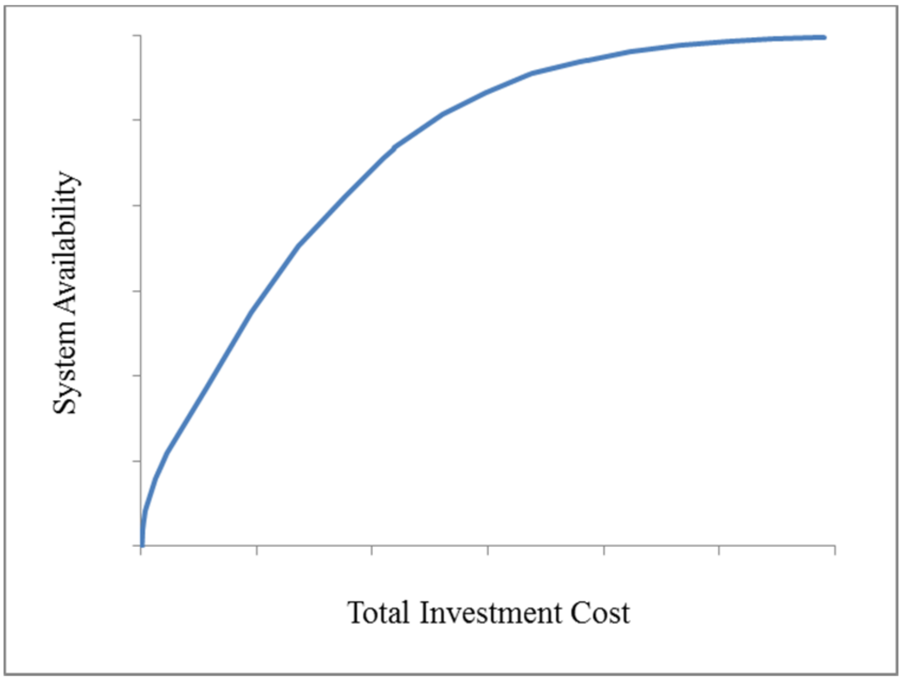


Figure 2. System Availability versus Total Investment Cost Curve.

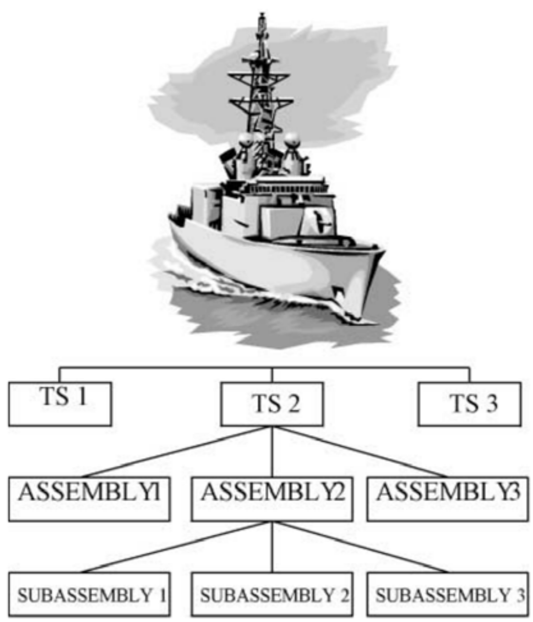
2. MOD-METRIC

The METRIC model has been used as benchmark by several researchers to derive several system approach models. Muckstadt (1973) developed an extended model so-called MOD-METRIC, which was implemented by the USAF to compute optimal spare stock levels in support of F-15 weapon system.

MOD-METRIC was developed based on METRIC system, though allowing the application of the optimization method in a multi-indenture environment. Hence, while

the latter disregards the hierarchy of parts structure, the former considers the logistics impacts caused by the existing relationship between an assembly and its components (p. 472). An example of system parts structure with two indenture levels is shown in Figure 3.

MOD-METRIC is considered the first multi-item, two-echelon, two-indenture system. Muckstadt (1973) observed, however, that the model can be easily extended to more echelons and indentures as well. Even though MOD-METRIC is considered the first multi-indenture, multi-echelon model, Van Houtum and Kranenburg (2015) noted that Sherbrooke (1971) addressed the existence of multi-indenture levels, yet for a single site system problem (p. 181).



This figure illustrates the material breakdown of a frigate's technical systems (TS). The assemblies represents the TS first indenture level and the subassemblies the second indenture level.

Figure 3. System Parts Structure. Source: Rustenburg, van Houtum and Zijm (2001).

In describing the conceptual system structure of MOD-METRIC, Muckstadt (1973) observed that new aircraft engines were designed to be more modular than

previous engines. He stated that this new design increased the number of engines in a serviceable condition, by reducing its repair time, given that it requires just to remove the defective module from the engine, replacing it by a serviceable module from stock. Muckstadt explained that almost all maintenance tasks would occur in the defective module since “the vast majority of recoverable parts are located in modules” (Muckstadt, 1973, p. 473).

Regarding the resupply time, Muckstadt (1973) explained that its duration depends on the location of maintenance (base or depot) and type of repair. He asserted that the modularity concept means that items have different criticalities, since a backorder on a module has a different logistical impact, when compared to a backorder on an engine. See Figure 4 for the module repair conceptual system utilized by Muckstadt to develop MOD-METRIC model.

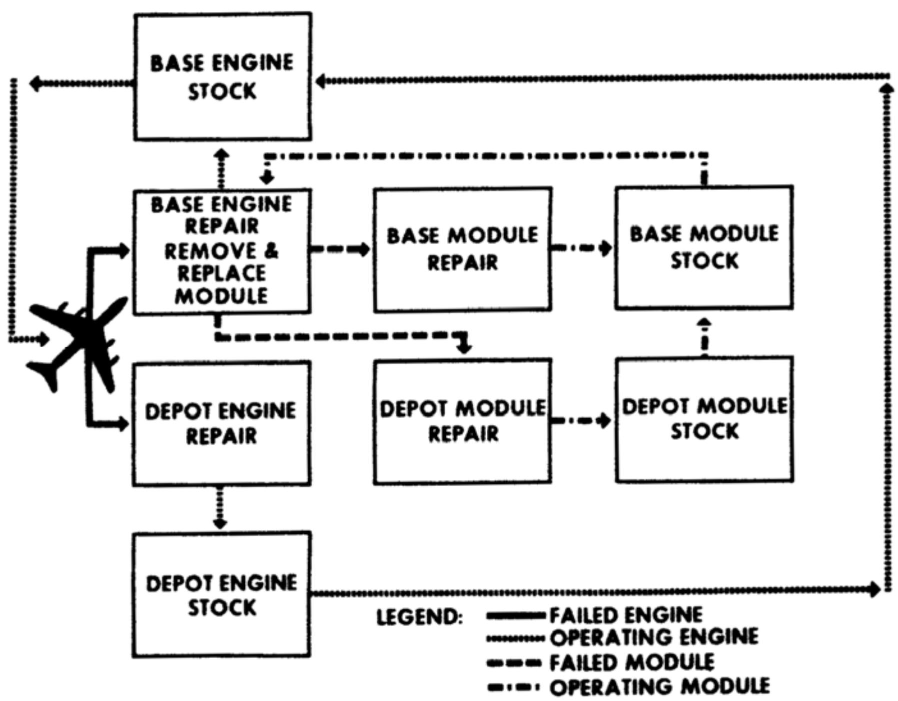


Figure 4. Module Repair Concept. Source: Muckstadt (1973).

With regard to the assumptions adopted to develop the inventory model, since MOD-METRIC focuses on the multi-indenture structure of end items to demonstrate the

logistics impacts of a part and its components, it clearly does not utilize the *equal essentiality* assumption used for METRIC. In fact, the main difference on the assumptions adopted by those models lies on this assumption. Muckstadt expressed the relationship between engine and modules in the equation that represents the average resupply time for an engine. Thus, MOD-METRIC has a different objective as well which is “to minimize the expected base backorders for the end item subject to an investment constraint on the dollars allocated to the end item and its components” (Muckstadt, 1973, p. 475).

3. VARI-METRIC

VARI-METRIC, another extension of METRIC, originally presented by Slay (1984), takes into account the variance of the number units in pipelines to make the calculations. He made an improvement to METRIC, adopting a negative binomial distribution for the number of units in repair, in a two-echelon, single-indenture system. Prior METRIC models consider only the mean, ignoring variance, to make pipeline calculations.

In a similar vein, Graves (1985) devised a general framework for reparable items in a single-indenture, two-echelon system, consisting of one depot and several bases. Graves developed an exact model to find the steady state distributions of net inventory levels at the bases, assuming that failures follow a compound Poisson process and deterministic resupply time from depot to the bases. With regard to the repair process at depot, no assumptions were used, though. In explaining that the computations of steady state distributions of the exact model are complicated, Graves proposed an approximate model to ease the computational burden of the exact model (p. 1251).

In the approximate model, Graves used the assumptions of Poisson process for the failure rate and ample repair capacity at the depot. Furthermore, he suggested that when the failure demand process is Poisson, the number of units in resupply at a base is unimodal, has a variance always greater than its mean, and could, therefore, be approximated to a negative binomial distribution (p. 1252). He then compared this approximate model with the METRIC, using the exact model results, on a set of test

problems do determine stock levels at the bases. The results indicated that in 11.5 % of the cases, the stock levels at the bases calculated using METRIC model differed from the solutions given by the exact model, whereas the negative binomial distribution approximate model differed only in 0.9% (Graves, 1985, p. 1253). Graves therefore concluded that, while both approximations are extremely effective, the negative binomial approximate model outperformed the METRIC (p. 1253).

In assessing the results of this comparison, Sherbrooke (1986) observed that even more important than Graves' findings, was the significant improvement of VARI-METRIC, adding that the simple derivation method developed by Graves makes the calculations of stock levels much easier than previous improvements to METRIC model (p. 311).

Following the work of Slay and Graves, Sherbrooke (1986) extended the VARI-METRIC model to incorporate multi-indenture problems. He again utilized the assumption of compound Poisson for failure process, as well as the other assumptions adopted for the original METRIC model, described in this chapter. Similarly to the work of Graves, the assumption of negative binomial distribution was adopted to approximate the number of items in resupply or repair, meaning that the variance is greater than its mean and unimodal.

Moreover, through the use of simulation to compute backorders, he demonstrated that VARI-METRIC, applied to a combined multi-echelon and multi-indenture problem, obtain extremely accurate results. His simulation also showed that MOD-METRIC, in comparison to VARI-METRIC and to the true number of backorders, always underestimates the expected backorders, except when stock levels of reparable items at the bases are equal to zero (Sherbrooke, 1986, p. 318).

Therefore, Sherbrooke affirmed that models that do not take into account pipeline variances, such as METRIC and MOD-METRIC, when used in a multi-item application based on backorder (or system availability) performance measures, underestimate the budget requirements, given that the expected backorders are also underestimated (Sherbrooke, 1986, p. 318). Furthermore, he asserted that "VARI-METRIC computation

is only slightly more complicated than that of METRIC, since it requires an estimate of the variance as well of the mean of backorders at each stage and the use of a binomial distribution rather than a Poisson" (p. 319).

4. Dyna-METRIC

Dyna-METRIC is another variant of METRIC. It is a multi-echelon as well as multi-indenture mathematical model. The prefix "Dyna" refers to the term *dynamic*, meaning that the new model can deal with a dynamic environment. Hillestad (1982) affirmed that one key characteristic of Dyna-METRIC "is its ability to deal with the dynamic or transient demands placed on component repair and inventory support" (Hillestad, 1982, p.5). This is a crucial distinction to the previous METRIC models described in this chapter, which utilize the assumption of *stationary demand* as the starting point to perform mathematical the computations for steady state cases.

Several versions of Dyna-METRIC were published. The earliest version was officially reported by Hillestad (1982). He emphasized, however, that the work of Hillestad and Carrillo (1980) was vital to the development of Dyna-METRIC, as they provided the theoretical development and the nonstationary pipeline equations for the expected number of items in the repair, or resupply, for each echelon. The most recent version of Dyna-METRIC was published by Isaacson and Boren (1993).

The new model was clearly designed for nonstationary situations, in particular, to consider wartime scenarios. Dyna-METRIC is result of the development of new analytic methods to examine the "transient behavior of component repair/inventory systems under time-dependent operational demands and logistics decisions like those that might be experienced in wartime" (Hillestad, 1982).

Another major characteristic distinguishing Dyna-METRIC from other METRIC extensions is the ability to handle *cannibalization,* described as the practice of removing a serviceable component from one unserviceable aircraft, to repair another aircraft, due the lack of spare parts (Isaacson & Boren, 1988). Hillestad and Carrillo (1980) addressed problems for different degree of cannibalization. In explaining the reason for incorporating this feature into the model, they say that the objective was to "suggest

approaches to certain aspects of recoverable item repair and supply which currently cause significant deviation between practice and theory, even in the case of stationary demands and service rates" (p. 2).

Moreover, Dyna-METRIC was designed as an analytical tool to measure readiness effects of maintenance supply chain processes. Hillestad described it as a model "a mathematical model for relating aircraft spare-parts supply levels and maintenance capability to material readiness of aircraft" (Hillestad, 1982). He continued on explaining that the model was "developed to study and predict the readiness of groups of aircraft squadrons as determined by a major subset of logistics resources…associated with component repair and resupply" (p. 2).

Other benefits of Dyna-METRIC were described in the implementation report of Pyles (1984). In explaining the reasons to the logisticians for the need of a new model, he outlined the following new type of information to support decision:

1. Operational performance measures
2. Effects of wartime dynamics
3. Effects of repair capacity and priority repair
4. Problem detection and diagnosis
5. Assessments or requirements (Pyles, 1984, p.3)

Despite of new capabilities brought by Dyna-METRIC, it has one significant drawback. The latest version of the model is described as a "capability assessment model" which means that it is not able to calculate spare parts requirements (Isaacson and Boren, 1993). Sherbrooke (2004) also highlighted this limitation. He stated that "Dyna-METRIC is best described as an assessment model rather than an optimization model" (Sherbrooke, 2004, p. 194). He added that only when used in a single echelon and single indenture case, the model provides optimal solutions.

5. Other METRIC-Based Models

While a large number of other METRIC based models had been developed over the last decades, it is worth mentioning two models, currently been used in the military

services to manage reparable inventories. These are the Aircraft Availability Model (AAM), presented by O'Malley (1983), and the Aircraft Sustainability Model (ASM), developed by Slay and King (1987). They are based, respectively, on VARI-METRIC and Dyna-METRIC models.

Both the AAM and ASM were developed for a multi-indenture multi-echelon (MIME) system and have several common assumptions with the models that they are based on. In fact, the ASM is also an extension of the AAM, with a major distinct feature of being capable to consider wartime scenarios when determining optimal spare parts support.

This section, however, does not intend to make a detailed description about the concerning models. Though, it is important to highlight that they focus on availability performance measure, as the models names suggest, to develop the theoretical framework. O'Malley argues that fill rate and backorders are not adequate performance measures to target when trying to maximize readiness, because it is unclear "what constitutes an acceptable fill rate or backorder level" (1983, p. 1–1).

C. GENERAL UNDERLYING MATHEMATICAL CONCEPTS

1. The $(s-1, s)$ Ordering Policy

Expensive items with low demand pattern are frequently managed by the (s - 1, s) policy, also known as one-for-one ordering policy, where s represents the stock level. Reparable items are typically included in this category of items. That is, whenever a demand occurs, a repair or resupply of one unit is immediately triggered. Models based on METRIC system utilize the (s - 1, s) policy to compute the optimal stock levels (Gross, 1982).

The lot size of one single unit for this class of items can be easily demonstrated by a simple observation of the classical Economic Order Quantity (EOQ) formula, developed by Harris (1913)

$$EOQ = \sqrt{\frac{2KD}{ic}}, \tag{2.1}$$

where:

EOQ = the economic order quantity in units

K = the cost to place an order

D = the annual demand in units

i = the annual holding rate cost per unit per year

c = cost per unit

Assuming that the cost to place an order is insignificant when compared to the unit cost of the item, costly and low demand items produce a numerator of Equation (2.1) lower than its denominator. Since it is not possible to order fractions of an item, order quantity is rounded to an integer number, which, in this case, is one.

Having defined the lot size, the challenge is then to calculate the appropriate stock levels and locations. An interesting implication of this policy is that inventory position remains constant. This happens because the order quantity equals the demand (Muckstadt, 2004, p. 36). The inventory position is represented according to the following equation

$$s = OH + DI - BO, \tag{2.2}$$

where:

OH = number of units on hand

DI = number of units due in

BO = number of units backordered

2. Ample Repair Capacity

The *ample repair capacity* (infinite channel assumption) is a key assumption employed for the development of METRIC models (Gross, 1982). This assumption means that failed items do not queue up, leading to statically independent repair lead times, as well as equal mean and variance for the number of items in repair process (Sleptchenko, Van der Heijden, & Van Harten, 2002).

Although the ample repair capacity assumption seems unrealistic, given that in fact the repair time is affected by the queues observed in practice, Sherbrooke explained that when a critical item fails, making a weapon system unavailable for a mission, corrective maintenance actions are prioritized to recover the failed item to its serviceable condition. Therefore, although the assumption understates the repair time delay, the expedition of corrective maintenance overstates the repair lead time (Sherbrooke, 2004).

Furthermore, he argued that while these factors tend to offset each other, the net result probably understate the real repair time. He then asserted that the "assumption of independent repair times is a reasonable approximation, and this is reinforced by over forty years of usage" (Sherbrooke, 2004, p. 23). Thus, the repair time then depends solely on the type and the complexity of repair.

It should be noted that other researchers have provided more complex models based on limited capacity repair capacity or closed queueing network, instead of the ample repair capacity, such as the works presented by Gross (1982), Isaacson and Boren (1988), Albright (1989), Diaz and Fu (1997), Sleptchenko et al. (2002), and Lau and Song (2008).

3. Palm's Theorem

The remarkable Palm's theorem (Palm, 1938) plays a very important role in the development of optimization models, including the METRIC models presented in this chapter. Under the assumption of *ample service capacity*, Palm's theorem, in repair process terms, states:

> If demand for an item is given by a Poisson process with annual mean *m* and the repair time for each failed unit is independently and identically distributed according to any distribution with mean *T* years, then the steady state probability distribution for the number of units in repair has a Poisson distribution with mean *mT*. (Sherbrooke, 2004, p. 22)

That is, the steady state probability for the number of units in repair is

$$P(x) = \frac{e^{-mT}(mT)^x}{x!} \qquad x = 0, 1, 2, \ldots \tag{2.3}$$

where:

P(x) = probability of units in repair

x = number of units in repair

m = mean annual demand

T = mean time period in years

In short, if demand arrival follows a Poisson distribution, the steady state is also Poisson, regardless of the distribution function of repair service. Sherbrooke highlighted the relevance of this aspect, observing that, therefore, it is not necessary to "collect data on the shapes of the repair distributions" (Sherbrooke, 2004, p. 22).

Palm's theorem is useful to find other steady state distribution shown in Equation (2.2). Graves (2006) showed how to derive the distributions of both the stock on hand (OH) and the backorders (BO) from the steady state distribution of the number of failed items (p. 153). Feeney and Sherbrooke (1966) also demonstrated how the steady state distributions OH and BO can easily be captured from the steady state distribution for the number of units in resupply.

Various researchers showed that Palm's theorem can be generalized to other situations. For example, Feeney and Sherbrooke (1966) provided an extension to compound Poisson demand, concluding that "Palm's theorem can be generalized to any compound Poisson distribution" (p. 392). Another interesting utilization of Palm's theorem refers to its extension for non-stationary cases. Hillestad and Carrilo (1980), in their mathematical report used for Dyna-METRIC, derived Palm's theorem to both nonstationary demand and service processes (p. 5). The theorem is also extended to finite populations as shown in Sherbrooke (1966).

III. CASE STUDY

A. INTRODUCTION

This chapter presents a case study of a Brazilian naval aircraft, by exposing a readiness-based METRIC model to determine optimal stock levels of a selected group of critical reparable components concerning the analyzed aircraft. The chapter also uncovers the theoretical development of the model, data requirements and the optimization algorithm.

B. BACKGROUND OF THE BN MAINTENANCE SUPPLY SYSTEM

This section is intended to provide the reader with a quick background view of how the BN supply system is structured and to briefly describe the current inventory management information system employed to control spare parts inventories.

1. The BN Supply Support Structure

Organizationally, the BN maintenance system is no different from typical military structures. The network comprises various types of organizations, each one responsible for the execution distinct and interdependent functions necessary to support highly technological weapon systems.

The Brazilian Navy Supply System (BNSS) is directly tied to the maintenance function, as stated in the BN directive EMA-400 (Estado-Maior da Armada [EMA], 2003, p. 2–2). The BNSS is defined as the set of organizations, processes and resources, interconnected and interdependent, structured in order to promote, maintain and control the provision of material necessary for maintenance of naval forces and other naval organizations in full efficiency conditions (Secretaria-Geral da Marinha [SGM], 2009). Different types of materials flow through the BNSS, including spare parts. The BNSS serves as the interface between the suppliers and the retail organizations.

Concerning the ability to perform maintenance activities, conceptually the BN encompasses a four-echelon environment (EMA, 2003, p. 4–8). Military operational units, such as ships, submarines and squadrons are part of the first-echelon, characterized

by simpler maintenance tasks. The second-echelon and third-echelons are constituted by the Naval Bases, equipped with greater maintenance resources. The fourth-echelon corresponds to the maintenance resources that exceed the BN capabilities, and therefore are performed by the original manufacturers, or authorized contractors.

Spare parts are accumulated in those echelons to allow the quick execution of maintenance tasks. The main reason is to minimize the waiting time for spare parts, reducing the system's downtime, to increase its availability. Conceptually, as stated in (SGM, 2009, p. 1–12), the BN has two types of spare parts allowance lists:

- Shipboard allowance list, corresponding to the material stocked in the organizational unit to support the first echelon tasks
- Base allowance list, corresponding to inventories that should be carried on distribution centers to support the second and third echelon maintenance activities

The same directive also determines that the BN technical directorates, also responsible for establishing the quantities for initial provisioning, shall periodically adjust allowance lists based on real demand data.

The management of spare parts inventory levels, corresponding to the wholesale level, occurs at the Brazilian Navy Inventory Control Point (BNICP). A wide and diverse range of systems are supported by the BNICP, including maritime and aviation weapon systems components. Prices and demand pattern are also highly diversified.

2. The BN Supply Management Information System

The BNSS is supported by a Management Information System so-called SINGRA, implemented in 2001. Among other features, SINGRA supports inventory management of spare parts and other categories of items such as fuels, uniform, office supplies and so on. Regarding spare parts, approximately 600,000 stock keeping units (SKU) are registered in the database.

SINGRA adopts the item-oriented approach to compute inventory requirements, as mentioned in Chapter I. The computation relies heavily in historical data to determine lot sizes, reorder points and safety levels. The following classical forecast demand

methods are embedded in SINGRA, to support management decisions: moving average, exponential smoothing and linear regression. These methods are available to all categories of supply and consider the following input data: historical demand, lead time and a target service level. Service Level requirements are established according to the category of supply item. For spare parts, the target service level is fixed on 80%.

Expensive items with stochastic demand have their replenishment planned on a one-to-one basis, which means that the procurement order is triggered only when a demand occurs, as described in Chapter II. This task is assigned to the BN technical directorates.

It is important to emphasize that the system was not developed to support exclusively the maintenance supply chain. Thus, it was not appropriately customized for the specificities concerning the management of spare parts.

C. DATA GATHERING

The utilization of reliable input data is fundamental to the accuracy of the case study results. Data gathering, however, proved to be a huge challenge in this research. As explained in the previous section, SINGRA was not tailored to the management of spare parts. Therefore, unfortunately it lacks essential required data for the utilization of a system approach optimization model.

For example, logistics information systems usually incorporate data about the reliability and maintainability of reparable components, in particular, the mean time between failures (MTBF) and mean time to repair (MTTR). These are crucial parameters for the management of reparable inventories. Other important data for the RBS model, such as criticality and breakdown structure, are also not available in the System database.

Consequently, to allow the development of the case study, it was necessary to collect information from other sources. Part of the data was collected directly from the operational level unit, in particular from the unit's maintenance and repair sector, as described further in this chapter.

1. Weapon System Selection

To overcome data limitation issues, choosing a proper weapon system to be analyzed in this case study was a crucial factor to ease accurate data gathering. For various reasons, a naval aviation asset, rather than a naval surface asset, was the selected for this pilot case study.

The first reason concerns the reliability of components data. In the maritime context, usually little historical failure data is collected, which hampers adequate demand projections (Eruguz, Tan, & van Houtum, 2015). In addition, naval assets systems are more subject to variations in failure rate, given the environmental conditions present in the maritime sector (Eruguz et al., 2015). On the other hand, the aviation community typically keeps strict control of failure data records for each critical component of the aircraft.

Another reason for selecting an aircraft is related to equipment usage. Fluctuation on equipment usage naturally affects the demand rate. Aircrafts, in general, are assigned to relatively stable operation missions, while ships are more subjective to variable deployment scenarios. Maritime assets, for example, frequently deploy for long duration missions (Eruguz et al., 2015).

Modularity was also considered when selecting an aircraft for the case study. This feature is associated to maintainability. In general, aircraft systems are more modular designed than the technical systems of ships. Therefore, the corrective maintenance time is less variable and easier to estimate.

It is important to stress that even though this research does not focus in demand forecasting, the utilization of reliable demand data is a major factor to the accuracy of the model results. Therefore, due the above-mentioned aspects, an aircraft was chosen to be analyzed in this case study. The selected aircraft is the Bell 206 Jet Ranger III single turbine helicopter.

2. Field Setting

The Bell 206 Jet Ranger III helicopter, selected for this case study due the reasons previously exposed, is assigned to a squadron located at the Brazilian naval air base in Rio de Janeiro state. Both the squadron and the naval air base count with skilled personnel to perform preventive and corrective maintenance on Bell 206 Jet Ranger III helicopters.

The squadron performs the first echelon maintenance and repair tasks, while the naval air base is responsible for the second and third echelon maintenance activities. In addition, in regard to the analyzed helicopter airframe, the naval air base has extensive repair and overhaul capabilities, which is typically related to the fourth-echelon maintenance level. In contrast, engine's critical components are repaired outside the BN, by the engine's manufacturer authorized contractor.

If on the one hand there is more than one echelon of maintenance, on the other hand, from an inventory control perspective, the model is considered a one-echelon system, given that the reparable components of the selected type of helicopter are stocked in one single site, located inside the base.

When a failure occurs on the helicopter, the maintenance personnel from the squadron unit remove the failed component from the helicopter, replacing it by a serviceable unit from stock, thereby returning the helicopter to its operationally ready condition. These items, replaceable at the operational level, are known as line replaceable units (LRU), defined as "an essential support item which is removed and replaced at field level to restore the end item to an operationally ready condition" (Department of Defense [DOD], 1991). LRU are also known as first indenture items.

If a serviceable spare is not available in stock to replace the failed LRU, a backorder is originated. Recall that a backorder causes a "hole" in the helicopter, which makes it not operationally available for a mission. The type of failure determines if the component will be repaired at the squadron, at the naval base, or by the authorized contractor.

While a helicopter contains a collection of technical systems, for the purpose of this case study, only the helicopter's engine and a group of its critical LRU will be analyzed. The following set of LRU was selected:

- Bleed Valve
- Governor
- Fuel Control Unit
- Fuel Pump
- Fuel Nozzle
- Compressor
- Gear Box
- Turbine

The major objective of the model is to compute optimal stock levels for the selected group of LRU, so that the availability of engines is increased at the least total investment cost. Maximizing the availability of engine is equivalent to maximize the helicopter availability, all other things equal.

Besides the optimization capability of the model, another useful application of the model is related to evaluation. The availability versus investment cost curve provides an assessment tool to compare different optimal inventory policies. Furthermore, it will be possible to contrast these policies with the results given by the current existing stock levels of the selected LRUs.

It is important to point out that, even though this study case is restricted to reparable parts, not encompassing consumable items, the model will still provide optimal solutions, just like the METRIC model case. The reason is that consumable parts, which are not in the scope of this case study, are characterized by relatively low cost and high demand, so that are be appropriately managed by the traditional inventory management approach, based on demand forecasting methods, for an established given protection service level, at the BNICP.

D. MODEL DESCRIPTION

1. Overview of Assumptions

This section describes the main underlying assumptions of the optimization model utilized to perform the computations.

(1) The (s - 1, s) ordering policy

The selected reparable items are sufficiently expensive to justify the utilization the (s - 1, s) policy, as explained in the literature review. This is a vital assumption to develop the mathematical expressions of the model.

(2) Items have equal essentiality

All the selected reparable components are considered equally critical to the aircraft; that is, a failure of any LRU is sufficient to put the engine in an unserviceable condition, which in turn makes the helicopter not operationally ready for a mission.

(3) Ample repair capacity

As discussed in Chapter II, this is another key assumption. It considers that the repair time for a LRU is not influenced by other LRU already in the repair process. That is the repair lead times for each LRU are statically independent.

(4) Stationary demand

The demand of a reparable item is typically stochastic, represented by a discrete random variable (Muckstadt, 2004, p.12). The selected aircraft for the case study is in the maturity phase of its life cycle. Thus, the demand for reparable items is assumed to be stationary, which means that the failure rate is constant and follows an exponential distribution. The oscillation of failure rate throughout a system life cycle is shown in Figure 5. The selected helicopter is in the “Useful life Period” section of Figure 5.

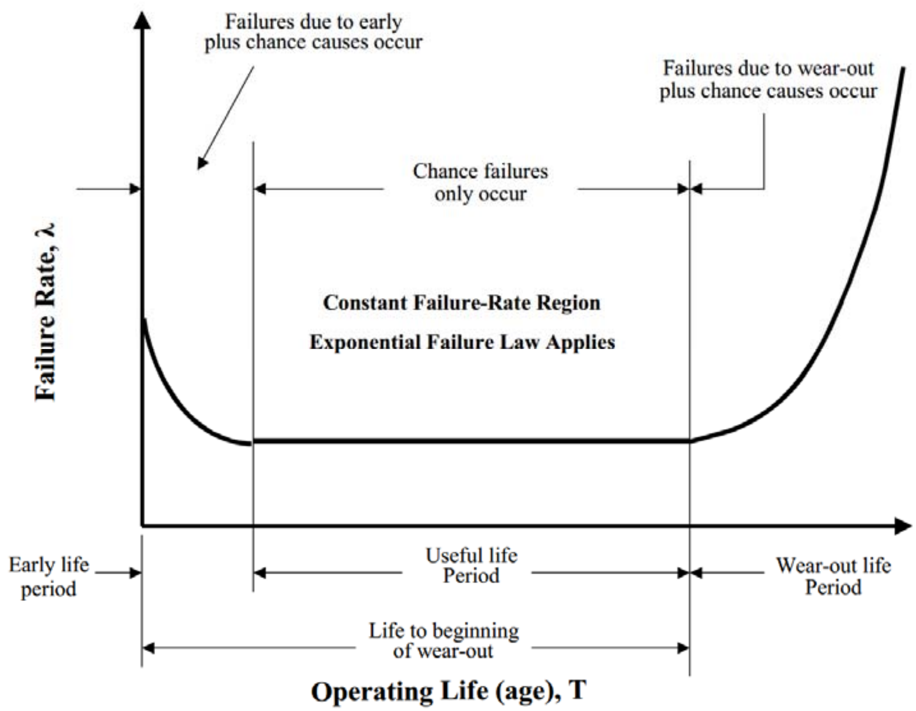


Figure 5. Reliability Bathtub Curve. Source: Kececioglu (2002).

(5) Poisson distribution

Poisson distribution is assumed to model the failure rate process, because at the present time, not enough failure data were available to fit the data to another analytical distribution. In fact, an estimate of the average of time between failures is the only available data. Poisson distribution is *memoryless*, meaning that the time period of the next failure is independent of the time elapsed since the last failure. Therefore, it seems appropriate to adopt the Poisson distribution in this case study to model the demand rate process, because this distribution requires only one parameter, the mean.

It is important to mention, though, that this is a conservative assumption, since Poisson distribution variance is equal to its mean. Therefore, the model will not consider the effects of higher values of variance for the number of units in repair, and this is possibly the case of the selected LRUs for the case study. As result, the model can underestimate the number of expected backordered demands and overstate the system availability accordingly.

(6) Cannibalization is not modelled

While in practice cannibalization of parts occasionally occurs, this action is often related to the lack of spare parts. Since the purpose of this case study is to determine reparable parts requirements as a matter of planning policy, cannibalization is not modelled.

(7) No lateral transshipments

Lateral transshipments are not modeled basically because the selected type of aircraft is present in one single squadron. Hence, the spares of the selected reparable components are kept in a unique stocking location. Furthermore, even if the selected reparable components are common to other type of aircrafts and therefore stocked in more than one stocking point, the effects of lateral transshipments are not in the scope of the case study.

(8) No disposal

The model assumes that failed components are not discarded, they can always be repaired. Again, data limitation hampers the possibility of modeling disposal rate. Research carried out by Weifenbach (1966) at Wright-Patterson Air Force Base, showed that disposals accounted for around only 4% of 10,965 maintenance actions. Thus, it seems reasonable to adopt the *no condemnation* assumption. Nevertheless, the model can be easily modified later to incorporate this feature.

2. Mathematical Expressions

The mathematical foundation of the case study relies heavily in the METRIC model described in the previous chapter, as well as in the theoretical development, and optimality proofs, given by Sherbrooke (2004), chapter two.

A major objective of the model is to maximize the availability of engines at the least cost. The target performance measure is clearly system availability. Recall, from the literature review, that the maximization of system availability is achieved through the minimization of total backorders (Sherbrooke, 2004). Thus, backorders can, in an

analogous way, be utilized as a target performance measure to develop the problem solution. This model will determine the optimal spare stock levels of the selected LRU from the expected backorders computations.

In fact, it is important to stress the importance of setting backorders as the main performance measure for the objective function of the problem. A backorder may be recorded as a two-dimensional variable that includes the moment that the demand occurs, and the duration of the shortage. Thus, backorder is directly related to system availability. In contrast, fill rate, the only performance measure embedded in SINGRA, does not capture the effects of lengthy unfilled demands.

At this point, it is important to recall the basic inventory position formula, as defined in Equation (2.2), where the stock level $s = OH + DI - BO$. Remember also that the inventory position remains constant due to the (s - 1, s) ordering policy and that a backorder is originated whenever there is no OH inventory to fulfill a demand (i.e., OH = 0).

Thus, from the basic inventory position equation and given that all the variables represent non-negative values, the size of the backorder is expressed as

$$BO\,(x \mid s) = (x - s) \text{ if } x > s \qquad (3.1)$$
$$= 0 \text{ otherwise}$$

where:

BO = backorders

x = random variable for the number of units in repair (DI)

s = target stock level

Equations (3.1) through (3.8) were developed by Sherbrooke (2004). Equation (3.1) shows that the number of BO units is a function of the number of LRU in repair and of the stock level. The number of units in repair, in turn, is modeled according to a Poisson process with annual demand m, and mean repair time T years.

Recalling Poisson distribution, Equation (2.3):

$$P(x) = \frac{e^{-m\mathrm{T}}(mT)^x}{x!} \quad x = 0, 1, 2 \ldots$$

The next step is to find the steady state distribution function of the expected backorder. This is a crucial value, because the optimization method seeks to minimize this performance measure.

Recall from Palm's theorem, described in the previous chapter, that the steady state distribution of expected backorders (EBO) for a given LRU can be calculated from the steady state distributions for the number of units in repair, denoted by *mT*.

$$EBO_i(s_i) = \sum_{x=s+1}^{\infty} (x - s_i)P(x \mid mT), \tag{3.2}$$

where:

$EBO_i(s)$ = expected backorders for the LRU_i with stock level s, where $i = 1,..., I$

x = random variable for the number of units in repair

The total number of backordered demands, denoted by EBO(S), is the sum of total expected backorders across all LRU stock levels. That is

$$EBO(S) = \sum_{i=1}^{I} EBO_i(s_i) \tag{3.3}$$

The total investment cost in spares is the sum of the investment costs in spares for each LRU

$$C(S) = \sum_{i=1}^{I} c_i s_i \tag{3.4}$$

where:

c_i = unit investment cost for the i^{th} LRU

The objective function is to minimize the sum of EBO(S), for a given budget constraint. The objective function can be conversely stated in terms of maximization of availability. Thus, after defining the EBO equations, attention is focused on the availability performance measure.

The term availability, in this context, means supply availability, representing the fraction of the aircraft fleet that is not grounded due the lack of LRU, at a random point in time, or equivalently the expected fraction of time that any helicopter in the squadron is not out of service (Sherbrooke, 2004, p. 38). Availability takes into account unscheduled maintenance action due to the occurrences of stochastic failures. It does not consider downtime caused by scheduled preventive maintenance.

Considering that the fleet squadron has *N* helicopters and that the total number of i^{th} LRU per helicopter is Z_i, the probability that the LRU is in working conditions equals to

$$1-\frac{EBO_i(s_i)}{NZ_i} \tag{3.5}$$

Therefore, under the assumption of independent failures and no cannibalization, the total availability of the system is given by the product of the availability of each LRU_i, as follows

$$A(S)=\prod_{i=1}^{I}\left(1-\frac{EBO_i(s_i)}{NZ_i}\right)^{Z_i} \tag{3.6}$$

Moreover, when targeting high levels of availability, it is demonstrated in Sherbrooke (2004) that by taking the logarithms in Equation (3.6), A(S) can be approximated to

$$A(S)\cong 1-\frac{EBO(S)}{N} \tag{3.7}$$

3. Inventory Setting Procedure

Having defined the key mathematical expressions, the next step is to determine the inventory policies. The method to calculate stock levels for the selected group of reparable items is based on a greedy heuristics.

It can be demonstrated that the objective function, EBO(S), is convex and decreasing. Furthermore, the problem is also separable. Sherbrooke (2004) provides proofs on the convexity and separability properties of the EBO function. A greedy

algorithm is therefore applicable to solve the problem. Furthermore, a range of optimal solutions can be generated through the utilization of the greedy method.

The greedy algorithm starts with stock level of zero for all LRUs. It then selects the first stocking unit, based on the item that provides the greatest contribution in terms of EBO reduction per unit investment cost. To put it differently, the method selects the item that yields the greatest "bang for the buck" system availability. The marginal decrease in EBO per unit investment cost is represented as follows

$$\Delta EBO_i / c_i = \frac{EBO_i(s_i) - EBO_i(s_i + 1)}{c_i} \tag{3.8}$$

Once the first LRU is selected, the greedy algorithm is repeated, selecting additional stocking units of i^{th} LRU using in Equation (3.6), until the system achieves a budget constraint. The mathematical objective is summarized as follows:

$$\min EBO(S)$$

Subject to: $C(S) \leq C^{obj}$

Alternatively, the optimization problem can be reformulated as minimizing the total investment cost, until the EBO(S) reaches a target EBO. This is achieved utilizing the same greedy algorithm described above. The mathematical problem, in this case, is stated as:

$$\min C(S)$$

Subject to: $EBO(S) \leq EBO^{obj}$

4. Input Data

The mathematical expressions in the previous section show that the model requires various input data elements to solve the problem and to generate the cost-availability curve. These data parameters are summarized as follows:

- Number of helicopters in the fleet
- Mean time between failures (MTBF)
- Average time spent in the repair process

- Flying hours per helicopter
- Unit investment cost
- Number of engines per helicopter
- Number of each LRU per engine

The squadron encompasses a fleet of sixteen Bell 206 Jet Ranger III helicopters (N=16), each one flying at a rate, on average, of 300 hours per year. These are single turbine helicopters. For all the selected LRUs, only one unit of each LRU is applied per engine (Z_i =1).

Data concerning the average time between failures was acquired directly by the maintenance unit of the squadron. The estimate of the average data was retrieved from an information system employed to control maintenance and repair actions of critical reparable components of aircrafts. Every failure of each individual reparable component is recorded in the database. This parameter is the MTBF, measured in operation hours.

Likewise, with regard to the average time spent in the repair process, data was also collected from the information system mentioned above, by the maintenance unit of the squadron. It is important to mention, however, that this value comprehends the whole time elapsed from the moment that the failed LRU is removed from the engine, until the moment it returns in a serviceable condition from the contractor. This value includes the time spent to fill the paperwork, to pack the item and the total shipment time. Recall that all the engines failed components are sent to an authorized contractor, outside the BN, in order to be repaired. The total time spent in this process can also be thought as turnaround time (TAT).

The required input data of the selected LRUs for the case study analysis is presented in Table 1. Furthermore, the current existing stock level for each LRU is shown in Table 2.

Table 1. LRUs Input Data

LRU	Description	MTBF (hours)	TAT (days)	Unit Cost
1	Bleed Valve	9,051	98	$ 3,987
2	Governor	5,352	78	$ 21,177
3	Fuel Control Unit	2,357	92	$ 33,634
4	Fuel Pump	8,707	105	$ 18,839
5	Fuel Nozzle	2,287	102	$ 5,581
6	Compressor	1,819	219	$ 151,870
7	Gearbox	2,035	211	$ 138,612
8	Turbine	2,876	259	$ 164,174

Table 2. Existing Stock Levels of LRUs

LRU	Description	Stock level
1	Bleed Valve	9
2	Governor	7
3	Fuel Control Unit	13
4	Fuel Pump	8
5	Fuel Nozzle	12
6	Compressor	9
7	Gearbox	7
8	Turbine	8

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IV. RESULTS AND ANALYSIS

A. INTRODUCTION

This chapter presents the results from the case study described in Chapter III. One major objective of this chapter is to present a range of optimal inventory levels of the analyzed LRUs and to generate efficient frontier curves (between the number of total expected backorders as well as average system availability, and inventory investment. MS Excel is used to perform the computations, to run the greedy algorithm, and to generate the efficient frontier curves. Furthermore, the current LRU inventories are entered in the equations previously depicted, so that the computed results can be compared to the optimal solutions given by the RBS model, to assess whether this approach could provide benefits if applied in the BN.

B. OPTIMAL SPARING REQUIREMENTS

1. Annual Demand

Having described the model, the main mathematical equations, and the input data from the previous chapter, it is crucial to estimate the mean annual demand m_i for each LRU. This data element is a function of usage rate, the forecasted flight hours for the next year. To estimate m, it is assumed that the helicopters will keep flying at the same rate observed in the past. Therefore, demand is projected as

$$m_i = \frac{NZ_i t}{MTBF_i} \tag{4.1}$$

where t = mean annual flying hours per helicopter in the fleet.

Recall that the squadron has 16 helicopters, each one flying, on average, 300 hours per year. Each helicopter has one unit of each LRU. Hence, the numerator of Equation (4.1) equals 4,800 annual flight hours. Data concerning the MTBF of LRUs was presented in Table 1. The mean annual demand m_i for each LRU is calculated accordingly. The results are summarized in Table 3.

Table 3. LRUs Average Annual Demand

LRU	Description	Average annual demand (*m*)
1	Bleed Valve	0.5303
2	Governor	0.8969
3	Fuel Control Unit	2.0365
4	Fuel Pump	0.5513
5	Fuel Nozzle	2.0988
6	Compressor	2.6388
7	Gearbox	2.3587
8	Turbine	1.6690

2. Average Pipelines

Once the annual demands of the LRUs have been projected, the steady state average number of units in repair or average pipeline stock (mT) can be calculated using the TAT data displayed in Table 1. Note that the TAT has to be previously converted to "years" since it was given in "days". The computed average pipelines are shown in Table 4.

Table 4. LRUs Average Pipelines

LRU	Description	Average annual demand (*m*)	T (in years)	Average pipelines (*mT*)
1	Bleed Valve	0.5303	0.2685	0.142389473
2	Governor	0.8969	0.2137	0.191657964
3	Fuel Control Unit	2.0365	0.2521	0.513306327
4	Fuel Pump	0.5513	0.2877	0.158587564
5	Fuel Nozzle	2.0988	0.2795	0.586519398
6	Compressor	2.6388	0.6000	1.583287521
7	Gearbox	2.3587	0.5781	1.363535391
8	Turbine	1.6690	0.7096	1.184293254
			Sum	**5.723576892**

3. Efficient Inventory Levels

Once all required input data have been presented, the efficient inventory levels are finally determined. The first optimal solution is produced when all stock levels equal

zero. The correspondent EBO(S) and A(S) are calculated, utilizing respectively Equations (3.3) and (3.6). This inventory level denoted as S_1, results in EBO(S_1) of 5.7236 and yields an availability of approximately 69% at no inventory investment cost. Note that before any LRU are stocked, the EBO(S_1) equals the total steady-state number of LRUs in repair.

The next efficient inventory levels are found by using the greedy algorithm explained in the previous chapter, selecting the stocking unit that provides the greatest marginal contribution in system effectiveness per investment cost, through the computations of $EBO_i(s)$ and $\Delta EBO / c$, by applying respectively Equations (3.2) and (3.8). These values, for stock levels from zero to five, are shown in Table 5. The spreadsheet of the remaining values computed to run the greedy procedure (stock levels 0–15) is presented in Appendix A.

Table 5. EBO and Marginal Benefit per Investment Cost (in thousands of dollars).

Stock level		0	1	2	3	4	5
LRU 1	EBO	0.14238947	0.00967288	0.00044831	0.00001573	0.00000044	0.00000001
	$\Delta EBO / c_1$	-	0.03328733	0.00231366	0.00010850	0.00000383	0.00000011
LRU 2	EBO	0.19165796	0.01724717	0.00106711	0.00005015	0.00000190	0.00000006
	$\Delta EBO / c_2$	-	0.00823586	0.00076404	0.00004802	0.00000228	0.00000009
LRU 3	EBO	0.51330633	0.11181975	0.01755390	0.00213722	0.00021180	0.00001767
	$\Delta EBO / c_3$	-	0.01193693	0.00280270	0.00045837	0.00005725	0.00000577
LRU 4	EBO	0.15858756	0.01193580	0.00061446	0.00002398	0.00000075	0.00000002
	$\Delta EBO / c_4$	-	0.00778448	0.00060095	0.00003134	0.00000123	0.00000004
LRU 5	EBO	0.58651940	0.14277944	0.02529678	0.00349224	0.00039339	0.00003736
	$\Delta EBO / c_5$	-	0.07950904	0.02105047	0.00390692	0.00055525	0.00006379
LRU 6	EBO	1.58328752	0.78858658	0.31893309	0.10660138	0.03007445	0.00730203
	$\Delta EBO / c_6$	-	0.00523277	0.00309247	0.00139811	0.00050390	0.00014995
LRU 7	EBO	1.36353539	0.61929037	0.22377633	0.06601579	0.01631703	0.00345478
	$\Delta EBO / c_7$	-	0.00536927	0.00285339	0.00113814	0.00035855	0.00009279
LRU 8	EBO	1.18429325	0.49025559	0.15856707	0.04144237	0.00901982	0.00167533
	$\Delta EBO / c_8$	-	0.00422745	0.00202035	0.00071342	0.00019749	0.00004474

Note in Table 5 that the second optimal inventory level (S_2) is found by selecting LRU 5. This first stocking unit generates an $\Delta EBO / c$ equals to 0.07950904, the largest value among all the LRUs. The resulting EBO(S_2) is 5.2798 and A(S_2) is 71%, at the investment cost of $5,581.

The optimal inventory level S_3 is obtained by choosing to stock LRU 1, which gives a marginal benefit of 0.03328733, subsequently followed by another unit of LRU 5, to form inventory level S_4, with the marginal value of 0.02105047. Note that stocking the second unit of LRU 5 provides greater system effectiveness than stocking the first unit of any other LRU that is yet to be selected.

Furthermore, the greedy procedure is repeated until the objective target is achieved. Since neither a target performance goal nor a C(S) constraint is yet established, with the purpose of limiting the presentation of optimal inventory levels, the greedy analysis is restricted to the values presented in Appendix A. Thus, the algorithm stops when the first LRU has its fifteenth unit selected, as the values presented in that Appendix are limited to stock levels 0–15. The resulting optimal inventory levels are exhibited in Appendix B. The correspondent EBO(S), A(S), and C(S) are shown in Appendix C.

4. Efficient Curves

After uncovering a range of the LRUs optimal inventory levels, the engine efficiency curves can be easily generated with the output data from these inventory levels. The engine EBO versus investment cost curve is displayed in Figure 6 and the engine expected availability versus investment cost curve is illustrated in Figure 7. Inventory levels S_1 to S_{32} were selected with the purpose of displaying the curves.

An interesting feature about the curves is that, in fact, a range of optimal points is exhibited in those figures, instead of a continuous curve. The optimal stock levels lie on what is called convex hull. The horizontal and the vertical distances between the adjacent points are given, respectively, by the cost and the marginal performance improvement of the additional spare LRU incorporated to inventory.

Moreover, eight different LRUs were selected for the case study. The higher the range of items included in the optimization model, the more the set of inventory points would tend to approximate a continuous curve. Nevertheless, the greedy procedure will never generate a continuous curve, since additional spares added to inventory levels are discrete numbers.

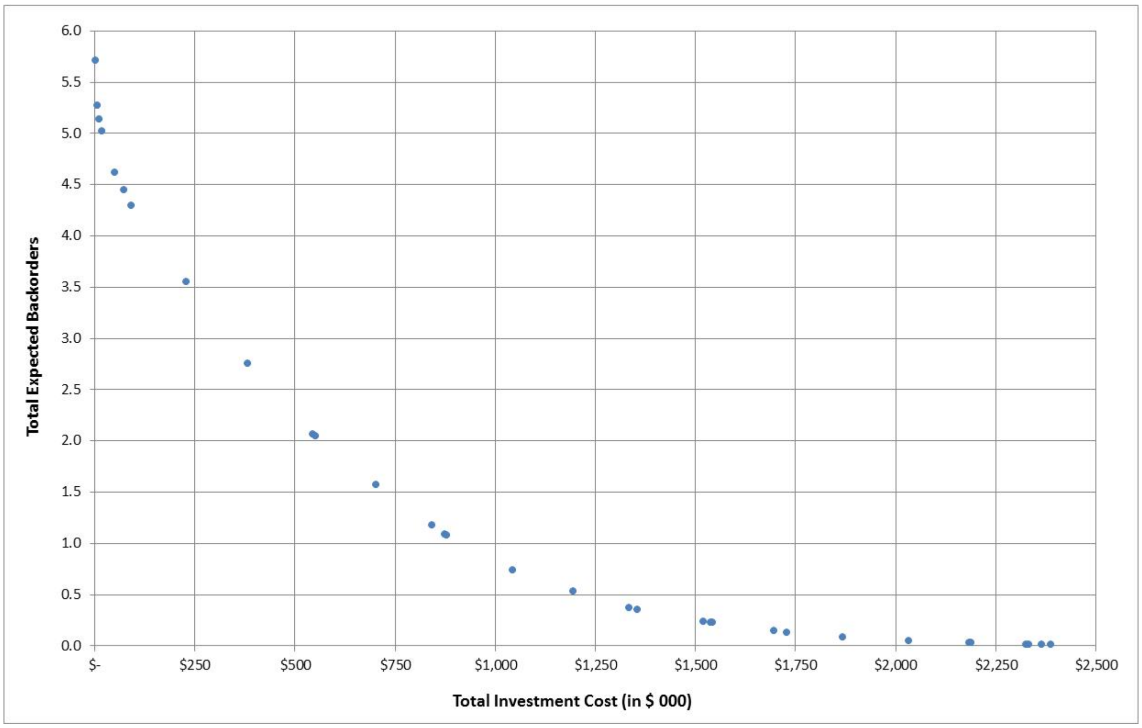

Figure 6. Engine EBO versus Investment Cost Curve

Also note that as greater system performance levels are demanded, lower are the system increments when additional spares are included in the inventory. For example, observe in Figure 7 that with an initial budget of $750,000 an expected availability of 90% can be achieved. This represents an incremental engine availability of approximately 20% when there are no spare in inventory. If additional $750,000 is incorporated to the initial budget, the expected availability increment will be around 7.5%. This illustrates what is known as the law of diminishing returns. Moreover, the efficiency curves show the existing relationship between system performance measures (EBO or Availability) and sparing investment cost.

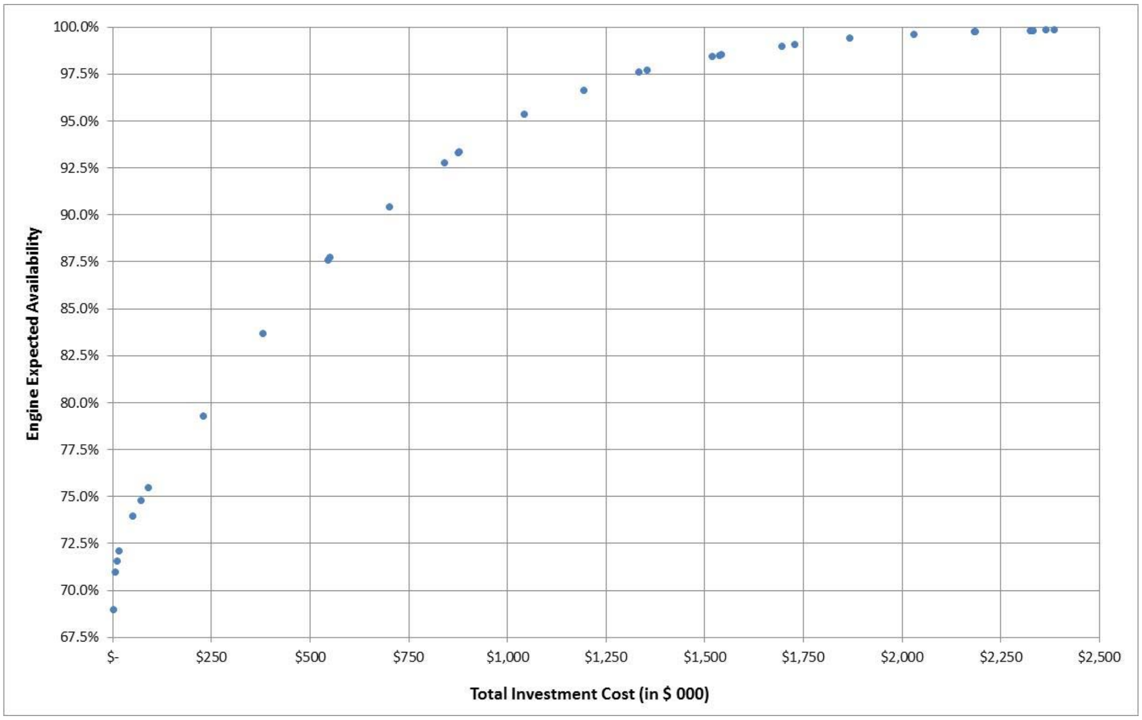

Figure 7. Engine Expected Availability versus Investment Cost Curve

C. EVALUATION OF EXISTING INVENTORY

Once the inventory levels and the efficient curves are presented, the evaluation of the LRUs current inventory is feasible. Availability will be used as the performance measure to compare existing stock levels of LRUs in the squadron with the computed sparing plans of the RBS model.

In order to make the comparison, the first step is to input the existing stock levels of each LRU, given in Table 2, into Equations (3.2), (3.4) and (3.6) to compute the projected availability and the total cost of the current inventory policy. The outputs are the following:

- Availability = 99.99927%
- Total Cost = $ 4,489,554

The estimated availability of the engine approaches 100%. This value indicates that whenever a demand occurs, spare LRUs are supposedly on hand (i.e., backorders are virtually zero). It should be emphasized that even with infinite stock levels, the engine

availability will never reach 100%, because a portion of time is required to remove the defective LRU and replace it by a serviceable one.

The current inventory point is then plotted together in Figure 8 containing the availability curve of the optimal inventory levels, to assess its position in relation to the efficiency curve. For ease of visualization, only a segment of the availability-cost curve is exhibited, with data of the optimal inventory levels set S_{47}–S_{59}, having the scales been adjusted accordingly.

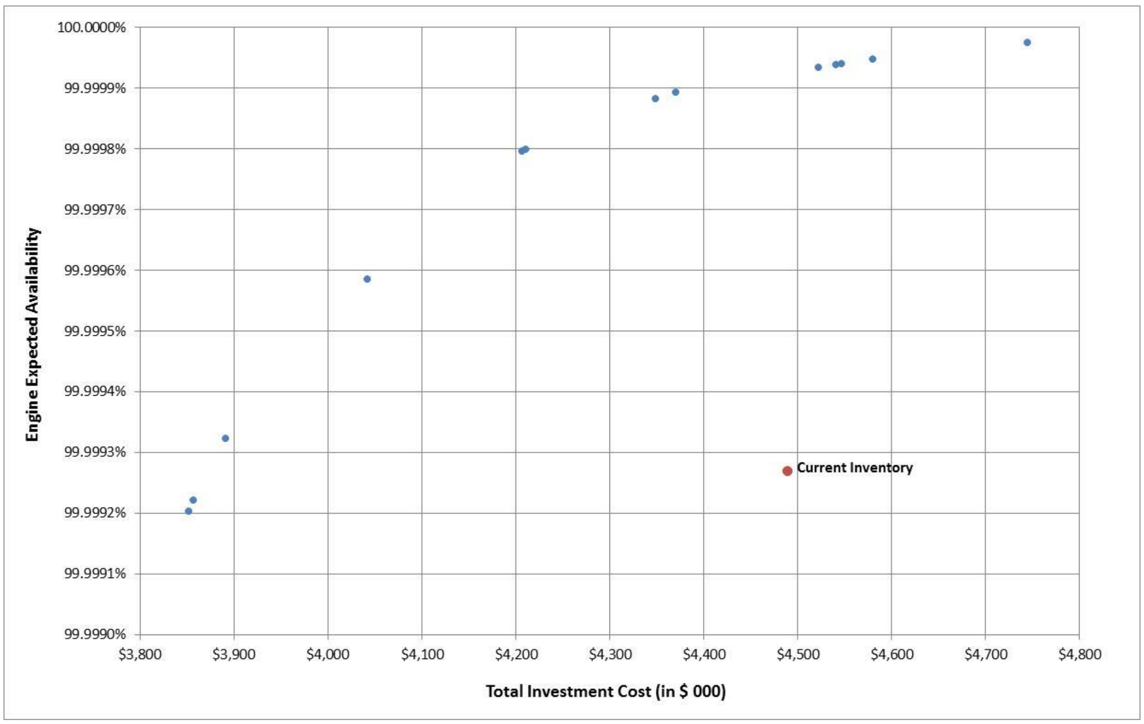

Figure 8. Current Inventory versus Optimal Inventory Levels

As seen in Figure 8, the current inventory, represented by the red dot, lies below the convex hull (i.e., the efficient frontier curve). This means that the current inventory policy is inefficient or sub-optimal.

1. Preliminary Efficient Alternatives

The graphical observation of Figure 8 also suggests that had a different sparing plan been picked from the convex hull, one of the following improvements would have been achieved:

- Maximized availability with a similar investment cost;
- Similar availability with the minimized investment cost; or
- A combination of the two above, that is better availability with less investment cost.

In particular, the inventory levels S_{54}, S_{49}, and S_{50}, selected from the efficient frontier address the above-mentioned points. These inventory levels, and the correspondent A(S) and C(S) are listed in Table 6.

Table 6. Example of Optimal Inventory Alternatives to the Current Inventory (availability goal nearly 100%).

LRU	**Description**	**Current Inventory**	**Optimal Inventory Alternatives**		
			S_{54}	**S_{49}**	**S_{50}**
1	Bleed Valve	9	5	4	4
2	Governor	7	5	4	4
3	Fuel Control Unit	13	6	6	6
4	Fuel Pump	8	4	4	4
5	Fuel Nozzle	12	7	7	7
6	Compressor	9	9	8	9
7	Gearbox	7	9	8	8
8	Turbine	8	8	7	7
Total LRUs		73	53	48	49
Availability		99.99927%	99.99989%	99.99932%	99.99959%
Total Investment Cost		$4,489,554	$4,369,777	$3,889,957	$4,041,827

These inventory alternatives indicate that excess inventory has been maintained, given that a similar or better performance level could be achieved with significantly reduced inventory levels. Nevertheless, by simply selecting one of the alternatives displayed in Table 6, or in Figure 7, one may still lead to inventory overstock. Targeting an availability of nearly 100% without regard the impact in sparing levels is unreasonable and costly, because of the diminishing marginal returns effect.

2. Meeting Operational Target

The selection of an optimal inventory level should be focused on a target availability goal. Unfortunately, the BN does not adopt the performance measure supply availability, as mentioned in Chapter III. This fact, however, does not hamper the examination of an improved inventory policy. LRUs should be kept in stock in enough quantity to meet consumer demands, in this case with the aim of reducing aircraft downtime. That is, availability goals should be in line with operational targets. The squadron has a planned operational availability (Ao) target of 75% for the helicopter fleet.

Therefore, bearing in mind that the model may overstate the system availability, as discussed in Chapter III, target availability goals of 85%, 90%, and 95% are now considered in the analysis, with the aim of illustrating sparing plans that would meet the operational target at lower investment costs. Observe in Appendix C that these goals would be achieved when the greedy procedure reaches, respectively, the optimal inventory levels S_{10}, S_{12} and S_{16}. The correspondent A(S), C(S) and LRUs stock levels, along with data of the current inventory, are displayed in Table 7. The percentage reductions of the selected inventory levels in comparison to the current inventory are summarized in Table 8.

Table 7. Proposed Optimal Inventory to the Current Inventory for Availability Goals of 85%, 90% and 95%.

LRU	Description	Current Inventory	Optimal Inventory		
			S_{10} (A=85%)	S_{12} (A=90%)	S_{16} (A=95%)
1	Bleed Valve	9	1	1	2
2	Governor	7	1	1	1
3	Fuel Control Unit	13	1	1	2
4	Fuel Pump	8	1	1	1
5	Fuel Nozzle	12	2	3	3
6	Compressor	9	1	2	2
7	Gearbox	7	1	1	2
8	Turbine	8	1	1	2
Total LRUs		73	9	11	15
Expected Availability		99.99927%	87.61985%	90.44841%	95.37553%
Total Investment Cost		$4,489,554	$ 543,455	$ 700,906	$1,193,183

Table 8. Percentage reduction in Comparison to Current Inventory

Percentage reduction	Optimal Inventory		
	S_{10}	S_{12}	S_{16}
Expected Availability	12.4%	9.6%	4.6%
Total Investment Cost	87.9%	84.4%	73.4%

As seen in Table 7 and Table 8, the existing inventory could be dramatically reduced to meet the depicted availability goals, and the LRUs allowances set accordingly, without theoretically compromising the Ao goal of the squadron. Note that despite of the large inventory reductions, the expected availability decreases are small, ranging only from 4.6% to 12.4%.

With regard to investment cost, it is important to point out that financial resources to purchase existing inventory have already been incurred. It is thus clear that reducing spares would not provide the cost savings shown for the values labeled as “Investment Costs”. Nevertheless, selling excessive inventories for a price below the investment costs is perfectly feasible and would provide some revenue to the BN, besides other benefits

such as reduced holding costs. Estimating these values though is not in the scope of this study.

In fact, the purpose of utilizing *investment costs* throughout the development of this Thesis, rather than other types of cost (e.g., holding cost) is to investigate significant costs that could have been avoided had the RBS model been in use by the BN.

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V. CONCLUSION

A. INTRODUCTION

This final chapter presents the thesis summary and major findings, proposes recommendations for the BN, points out limitations of the study and suggests areas for future work.

B. SUMMARY AND MAIN FINDINGS

The purpose of this research was to investigate a proposal to implement the readiness-based sparing methodology in the Brazilian Navy, to assess whether this approach could provide greater cost savings, without compromising the availability of weapon systems, in comparison to the traditional item-approach, which has been used to manage spare parts inventories, since the implementation of the existing inventory management system.

The examination focused on METRIC models, as tools based on these models have been widely employed by the military services for sparing purposes. Furthermore, a real-world case study was developed, using a group of critical components of a helicopter engine to explore the RBS theoretical concepts and to allow a quantitative comparison between existing inventory of these critical components and a range of efficient inventory levels proposed by the RBS approach.

The research showed that the RBS methodology is a useful analytical tool for inventory planning purposes, in a sense that it provides a range of efficient inventory alternatives, so that managers are able to make tradeoffs within this range and choose the most efficient plan to achieve a desired system performance measure at the least cost, or conversely to obtain the best performance for a given budget. By generating a tradeoff curve between spare inventory and system availability, the RBS method explicitly links the supply, operations and maintenance functions.

In particular, the case study illustrated one of the capabilities of the RBS tool: inventory assessment. The analysis provided the interesting indication that a portion of

inventory could be substantially reduced, in a prudent manner, without compromising the average availability of the supported equipment. Without the tool, it is tricky to identify the amount in excess and to determine stocks to be reduced, while maintaining the supported system availability goals.

The case study also showed that potential cost savings can be leveraged if the RBS methodology is applied to establish initial provisioning, in special to procure high cost reparable spare items. The utilization to this type of application seems to extract the most valuable benefit of the model. After making the initial investment in spares, if a larger than required amount was purchased, there is little room for cost savings.

Another major finding is the great dependence of the RBS model on a diverse data. Compared to the traditional item sparing approach, the system-approach is considerably more data intensive. As exposed in this thesis, required data to perform the quantitative system-approach analysis had to be collected from different databases. The available maintenance systems are used to control some data elements, but they lack mechanisms of collecting and sharing failure data. In sum, accurate data collection and database integration are possibly the greatest barriers for a comprehensive implementation of the RBS model in the BN.

C. RECOMMENDATIONS

Several indications exposed on this thesis indicate that the BN can take great advantages of the RBS. While this author recommends the implementation of the RBS methodology to the management of spare parts, in addition to the traditional demand forecasting method currently in use, it is important to point out that a thorough investigation is recommended prior to a comprehensive implementation of the model. A working group may be formed to this purpose, with representatives from logistics, engineering, technical, operational and IT areas of the BN.

The model developed for this thesis using MS Excel, along with the embedded greedy procedure, may be used as a starting point to extend the case study to a wide spectrum of weapon systems, as well as to apply to MIME inventory systems, considering that modifying the spreadsheets to allow the utilization to this context

should not be a difficult task. The advantage of using this tool is that MS Excel is readily accessible on virtually any personal computer. Therefore, there is no need of obtaining expensive commercial-off-the-shelf (COTS) RBS software to perform quantitative analysis during initial stage of the thorough investigation.

An eventual comprehensive implementation of the RBS model in the BN will certainly require great efforts, given the data barriers mentioned in the previous section. Data collection and databases integration will result on a challenging and long process. Therefore a phased implementation of the RBS methodology seems to be an interesting approach to be adopted.

In that sense, it is suggested to start applying the RBS tool for initial provisioning of reparables (and allowance lists determination) concerning the upcoming weapon systems to be procured by the BN, since this application provides the greatest benefit of the RBS methodology. Furthermore, this author acknowledges that estimating demand rates, as well as other input data, before operating the system is a difficult task, regardless of the estimation technique employed. Hence, one possible approach is targeting lower system availability targets, to form sufficient stock levels, until enough field data is gathered to allow more accurate estimates, so that complementary spares can be purchased if needed. This method would avoid high investment costs being tied to excess inventory, such as the scenario faced in the case study.

D. LIMITATIONS AND FURTHER RESEARCH

It is important to recall that several assumptions were adopted to run the model, as well as to overcome data availability issues. Poisson distribution was a fundamental assumption used to model the demand of all the LRUs, given that not enough data was available to fit other analytical distributions. Although Poisson has been extensively applied to model random discrete processes, it is important to reiterate that the variance of this distribution is equal to its mean, but the real-world data is probably over-dispersed around the average.

A major resulting limitation of this assumption, together with the other presented in this thesis, is that sparing solutions were computed focusing solely based solely on the

expected average system performance. That is, if on the one hand, from a tactical inventory planning perspective, the model is a useful analytical tool to determine sparing levels based on a ceiling on system availability, on the other hand, from the operational perspective, the model is inadequate to conduct readiness risk analysis, as the real observed system availability probably varies significantly.

To address the above-mentioned limitation, follow-up research may be conducted to gather enough data, in order to fit the demand and repair TAT of each LRU to its own particular analytical distribution. In addition, a risk analysis model shall be developed based on Monte Carlo simulation, to investigate the effects of variability on system readiness levels. In this respect, interesting studies on readiness risk analysis are found in Kang, Doerr and Sanchez (2006), and Doerr and Kang (2014).

Another theoretical drawback is related to the greedy algorithm. Note that the optimization problem described in this thesis is a type of *knapsack* problem. Thus, rather than finding all the optimal solutions of the problem, the greedy procedure yields what is best described as a set of efficient solutions. This means that an optimality gap exists between the targeted performance measure and the outcome of the efficient solution. The gap size depends on the unit cost of the last SKU added to inventory.

In fact, this heuristic approach was applied because finding exact solutions to *knapsack* problems is very complex and impractical. Moreover, for practical inventory problems with a large number of SKUs the greedy solution is good enough and robust (van Houtum & Kranenburg, 2015, p.23). Nonetheless, further research to find all the optimal solutions of the inventory problem may be conducted using a different mathematical method, such as dynamic programing, so that the set of efficient solutions can evaluated.

APPENDIX A. STOCK LEVELS, EBO AND ΔEBO / c_i

Stock level		0	1	2	3
LRU 1	EBO	1.42389E-01	9.67288E-03	4.48315E-04	1.57316E-05
	$\Delta EBO / c_1$	-	3.32873E-02	2.31366E-03	1.08498E-04
LRU 2	EBO	1.91658E-01	1.72472E-02	1.06711E-03	5.01505E-05
	$\Delta EBO / c_2$	-	8.23586E-03	7.64039E-04	4.80220E-05
LRU 3	EBO	5.13306E-01	1.11820E-01	1.75539E-02	2.13722E-03
	$\Delta EBO / c_3$	-	1.19369E-02	2.80270E-03	4.58366E-04
LRU 4	EBO	1.58588E-01	1.19358E-02	6.14458E-04	2.39751E-05
	$\Delta EBO / c_4$	-	7.78448E-03	6.00952E-04	3.13437E-05
LRU 5	EBO	5.86519E-01	1.42779E-01	2.52968E-02	3.49224E-03
	$\Delta EBO / c_5$	-	7.95090E-02	2.10505E-02	3.90692E-03
LRU 6	EBO	1.58329E+00	7.88587E-01	3.18933E-01	1.06601E-01
	$\Delta EBO / c_6$	-	5.23277E-03	3.09247E-03	1.39811E-03
LRU 7	EBO	1.36354E+00	6.19290E-01	2.23776E-01	6.60158E-02
	$\Delta EBO / c_7$	-	5.36927E-03	2.85339E-03	1.13814E-03
LRU 8	EBO	1.18429E+00	4.90256E-01	1.58567E-01	4.14424E-02
	$\Delta EBO / c_8$	-	4.22745E-03	2.02035E-03	7.13418E-04

Costs (c_i) are given in thousands of dollars and the values are displayed in scientific notation.

Stock level		4	5	6	7
LRU 1	EBO	4.43734E-07	1.04587E-08	2.11654E-10	3.75211E-12
	$\Delta EBO / c_1$	3.83443E-06	1.08672E-07	2.57011E-09	5.21451E-11
LRU 2	EBO	1.89765E-06	6.00591E-08	1.63304E-09	3.89130E-11
	$\Delta EBO / c_2$	2.27855E-06	8.67731E-08	2.75894E-09	7.52765E-11
LRU 3	EBO	2.11800E-04	1.76676E-05	1.27123E-06	8.03733E-08
	$\Delta EBO / c_3$	5.72462E-05	5.77191E-06	4.87494E-07	3.54063E-08
LRU 4	EBO	7.52355E-07	1.97345E-08	4.44542E-10	8.77309E-12
	$\Delta EBO / c_4$	1.23269E-06	3.88885E-08	1.02394E-09	2.31312E-11
LRU 5	EBO	3.93392E-04	3.73558E-05	3.06261E-06	2.20769E-07
	$\Delta EBO / c_5$	5.55250E-04	6.37943E-05	6.14464E-06	5.09199E-07
LRU 6	EBO	3.00745E-02	7.30203E-03	1.55138E-03	2.92460E-04
	$\Delta EBO / c_6$	5.03898E-04	1.49947E-04	3.78656E-05	8.28949E-06
LRU 7	EBO	1.63170E-02	3.45478E-03	6.38112E-04	1.04360E-04
	$\Delta EBO / c_7$	3.58546E-04	9.27932E-05	2.03205E-05	3.85070E-06
LRU 8	EBO	9.01982E-03	1.67533E-03	2.70789E-04	3.86896E-05
	$\Delta EBO / c_8$	1.97489E-04	4.47360E-05	8.55520E-06	1.41374E-06

Values are displayed in scientific notation and were computed with costs (c_i) given in thousands of dollars.

Stock level		8	9	10	11
LRU 1	EBO	5.90639E-14	6.66134E-16	0.00000E+00	0.00000E+00
	$\Delta EBO / c_1$	9.26272E-13	1.46470E-14	1.67076E-16	0.00000E+00
LRU 2	EBO	8.25146E-13	1.57929E-14	2.49800E-16	0.00000E+00
	$\Delta EBO / c_2$	1.79855E-12	3.82185E-14	7.33962E-16	1.17958E-17
LRU 3	EBO	4.53035E-09	2.30319E-10	1.06618E-11	4.52860E-13
	$\Delta EBO / c_3$	2.25495E-09	1.27848E-10	6.53081E-12	3.03530E-13
LRU 4	EBO	1.53988E-13	2.33147E-15	0.00000E+00	0.00000E+00
	$\Delta EBO / c_4$	4.57514E-13	8.05013E-15	1.23758E-16	0.00000E+00
LRU 5	EBO	1.41941E-08	8.23371E-10	4.35009E-11	2.10998E-12
	$\Delta EBO / c_5$	3.70140E-08	2.39576E-09	1.39737E-10	7.41639E-12
LRU 6	EBO	4.94916E-05	7.59189E-06	1.06441E-06	1.37363E-07
	$\Delta EBO / c_6$	1.59985E-06	2.75892E-07	4.29807E-08	6.10422E-09
LRU 7	EBO	1.52982E-05	2.03061E-06	2.46150E-07	2.74476E-08
	$\Delta EBO / c_7$	6.42524E-07	9.57172E-08	1.28738E-08	1.57781E-09
LRU 8	EBO	4.94895E-06	5.72713E-07	6.04875E-08	5.87365E-09
	$\Delta EBO / c_8$	2.05518E-07	2.66561E-08	3.12002E-09	3.32658E-10

Values are displayed in scientific notation and were computed with costs (c_i) given in thousands of dollars.

Stock level		12	13	14	15
LRU 1	EBO	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00
	$\Delta EBO / c_1$	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00
LRU 2	EBO	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00
	$\Delta EBO / c_2$	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00
LRU 3	EBO	1.76525E-14	5.55112E-16	0.00000E+00	0.00000E+00
	$\Delta EBO / c_3$	1.29395E-14	5.08338E-16	1.65045E-17	0.00000E+00
LRU 4	EBO	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00
	$\Delta EBO / c_4$	0.00000E+00	0.00000E+00	0.00000E+00	0.00000E+00
LRU 5	EBO	9.48130E-14	4.21885E-15	4.44089E-16	2.22045E-16
	$\Delta EBO / c_5$	3.61076E-13	1.62326E-14	6.76359E-16	3.97858E-17
LRU 6	EBO	1.64163E-08	1.82648E-09	1.90063E-10	1.85735E-11
	$\Delta EBO / c_6$	7.96386E-10	9.60678E-11	1.07751E-11	1.12919E-12
LRU 7	EBO	2.83294E-09	2.72105E-10	2.44366E-11	2.06024E-12
	$\Delta EBO / c_7$	1.77580E-10	1.84749E-11	1.78677E-12	1.61431E-13
LRU 8	EBO	5.27728E-10	4.41107E-11	3.44658E-12	2.52909E-13
	$\Delta EBO / c_8$	3.25625E-11	2.94576E-12	2.47689E-13	1.94529E-14

Values are displayed in scientific notation and were computed with costs (c_i) given in thousands of dollars.

APPENDIX B. OPTIMAL INVENTORY LEVELS

Inventory Level (S)	LRU 1	LRU 2	LRU 3	LRU 4	LRU 5	LRU 6	LRU 7	LRU 8	Investment Cost
1	0	0	0	0	0	0	0	0	$ -
2	0	0	0	0	1	0	0	0	$ 5,581
3	1	0	0	0	1	0	0	0	$ 9,568
4	1	0	0	0	2	0	0	0	$ 15,149
5	1	0	1	0	2	0	0	0	$ 48,783
6	1	1	1	0	2	0	0	0	$ 69,960
7	1	1	1	1	2	0	0	0	$ 88,799
8	1	1	1	1	2	0	1	0	$ 227,411
9	1	1	1	1	2	1	1	0	$ 379,281
10	1	1	1	1	2	1	1	1	$ 543,455
11	1	1	1	1	3	1	1	1	$ 549,036
12	1	1	1	1	3	2	1	1	$ 700,906
13	1	1	1	1	3	2	2	1	$ 839,518
14	1	1	2	1	3	2	2	1	$ 873,152
15	2	1	2	1	3	2	2	1	$ 877,139
16	2	1	2	1	3	2	2	2	$ 1,041,313
17	2	1	2	1	3	3	2	2	$ 1,193,183
18	2	1	2	1	3	3	3	2	$ 1,331,795
19	2	2	2	1	3	3	3	2	$ 1,352,972
20	2	2	2	1	3	3	3	3	$ 1,517,146
21	2	2	2	2	3	3	3	3	$ 1,535,985
22	2	2	2	2	4	3	3	3	$ 1,541,566
23	2	2	2	2	4	4	3	3	$ 1,693,436
24	2	2	3	2	4	4	3	3	$ 1,727,070
25	2	2	3	2	4	4	4	3	$ 1,865,682
26	2	2	3	2	4	4	4	4	$ 2,029,856
27	2	2	3	2	4	5	4	4	$ 2,181,726
28	3	2	3	2	4	5	4	4	$ 2,185,713
29	3	2	3	2	4	5	5	4	$ 2,324,325
30	3	2	3	2	5	5	5	4	$ 2,329,906
31	3	2	4	2	5	5	5	4	$ 2,363,540
32	3	3	4	2	5	5	5	4	$ 2,384,717
33	3	3	4	2	5	5	5	5	$ 2,548,891
34	3	3	4	2	5	6	5	5	$ 2,700,761
35	3	3	4	3	5	6	5	5	$ 2,719,600

Inventory Level (S)	LRU 1	LRU 2	LRU 3	LRU 4	LRU 5	LRU 6	LRU 7	LRU 8	Investment Cost
36	3	3	4	3	5	6	6	5	$ 2,858,212
37	3	3	4	3	5	6	6	6	$ 3,022,386
38	3	3	4	3	5	7	6	6	$ 3,174,256
39	3	3	4	3	6	7	6	6	$ 3,179,837
40	3	3	5	3	6	7	6	6	$ 3,213,471
41	3	3	5	3	6	7	7	6	$ 3,352,083
42	4	3	5	3	6	7	7	6	$ 3,356,070
43	4	4	5	3	6	7	7	6	$ 3,377,247
44	4	4	5	3	6	8	7	6	$ 3,529,117
45	4	4	5	3	6	8	7	7	$ 3,693,291
46	4	4	5	4	6	8	7	7	$ 3,712,130
47	4	4	5	4	6	8	8	7	$ 3,850,742
48	4	4	5	4	7	8	8	7	$ 3,856,323
49	4	4	6	4	7	8	8	7	$ 3,889,957
50	4	4	6	4	7	9	8	7	$ 4,041,827
51	4	4	6	4	7	9	8	8	$ 4,206,001
52	5	4	6	4	7	9	8	8	$ 4,209,988
53	5	4	6	4	7	9	9	8	$ 4,348,600
54	5	5	6	4	7	9	9	8	$ 4,369,777
55	5	5	6	4	7	10	9	8	$ 4,521,647
56	5	5	6	5	7	10	9	8	$ 4,540,486
57	5	5	6	5	8	10	9	8	$ 4,546,067
58	5	5	7	5	8	10	9	8	$ 4,579,701
59	5	5	7	5	8	10	9	9	$ 4,743,875
60	5	5	7	5	8	10	10	9	$ 4,882,487
61	5	5	7	5	8	11	10	9	$ 5,034,357
62	5	5	7	5	8	11	10	10	$ 5,198,531
63	5	6	7	5	8	11	10	10	$ 5,219,708
64	6	6	7	5	8	11	10	10	$ 5,223,695
65	6	6	7	5	9	11	10	10	$ 5,229,276
66	6	6	8	5	9	11	10	10	$ 5,262,910
67	6	6	8	5	9	11	11	10	$ 5,401,522
68	6	6	8	6	9	11	11	10	$ 5,420,361
69	6	6	8	6	9	12	11	10	$ 5,572,231
70	6	6	8	6	9	12	11	11	$ 5,736,405
71	6	6	8	6	9	12	12	11	$ 5,875,017
72	6	6	8	6	10	12	12	11	$ 5,880,598
73	6	6	9	6	10	12	12	11	$ 5,914,232

Inventory Level (S)	LRU 1	LRU 2	LRU 3	LRU 4	LRU 5	LRU 6	LRU 7	LRU 8	Investment Cost
74	6	6	9	6	10	13	12	11	$ 6,066,102
75	6	7	9	6	10	13	12	11	$ 6,087,279
76	7	7	9	6	10	13	12	11	$ 6,091,266
77	7	7	9	6	10	13	12	12	$ 6,255,440
78	7	7	9	7	10	13	12	12	$ 6,274,279
79	7	7	9	7	10	13	13	12	$ 6,412,891
80	7	7	9	7	10	14	13	12	$ 6,564,761
81	7	7	9	7	11	14	13	12	$ 6,570,342
82	7	7	10	7	11	14	13	12	$ 6,603,976
83	7	7	10	7	11	14	13	13	$ 6,768,150
84	7	8	10	7	11	14	13	13	$ 6,789,327
85	7	8	10	7	11	14	14	13	$ 6,927,939
86	7	8	10	7	11	15	14	13	$ 7,079,809

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APPENDIX C. EBO, AVAILABILITY AND INVESTMENT COSTS OF OPTIMAL INVENTORY LEVELS

Inventory Level (S)	EBO(S)	A(S)	Investment Cost
1	5.72357689181	68.9989269075%	$ -
2	5.27983693087	70.9853426899%	$ 5,581
3	5.14712033780	71.5794380503%	$ 9,568
4	5.02963768005	72.1097543556%	$ 15,149
5	4.62815110251	73.9791720051%	$ 48,783
6	4.45374030374	74.7953718604%	$ 69,960
7	4.30708854186	75.4877894783%	$ 88,799
8	3.56284352474	79.3262445064%	$ 227,411
9	2.76814258742	83.6989918269%	$ 379,281
10	2.07410492831	87.6198478043%	$ 543,455
11	2.05230038909	87.7394437919%	$ 549,036
12	1.58264689568	90.4484054878%	$ 700,906
13	1.18713284913	92.7742809336%	$ 839,518
14	1.09286699837	93.3247182010%	$ 873,152
15	1.08364243285	93.3785557474%	$ 877,139
16	0.75195390992	95.3755322558%	$ 1,041,313
17	0.53962219730	96.6669781900%	$ 1,193,183
18	0.38186166089	97.6336375535%	$ 1,331,795
19	0.36568160773	97.7324764370%	$ 1,352,972
20	0.24855690350	98.4550681020%	$ 1,517,146
21	0.23723555976	98.5247853395%	$ 1,535,985
22	0.23413671136	98.5438715912%	$ 1,541,566
23	0.15760978684	99.0183616359%	$ 1,693,436
24	0.14219310756	99.1138748205%	$ 1,727,070
25	0.09249434628	99.4230151296%	$ 1,865,682
26	0.06007180304	99.6250100142%	$ 2,029,856
27	0.03729938508	99.7670709375%	$ 2,181,726
28	0.03686680194	99.7697683601%	$ 2,185,713
29	0.02400455436	99.8500542031%	$ 2,324,325
30	0.02364851859	99.8522761447%	$ 2,329,906
31	0.02172309886	99.8642938464%	$ 2,363,540
32	0.02070613699	99.8706416559%	$ 2,384,717
33	0.01336164320	99.9165112211%	$ 2,548,891
34	0.00761099312	99.9524391736%	$ 2,700,761
35	0.00702051004	99.9561280793%	$ 2,719,600

Inventory Level (S)	EBO(S)	A(S)	Investment Cost
36	0.00420384048	99.9737283410%	$ 2,858,212
37	0.00279929964	99.9825053341%	$ 3,022,386
38	0.00154037501	99.9903729994%	$ 3,174,256
39	0.00150608180	99.9905873118%	$ 3,179,837
40	0.00131194929	99.9918005418%	$ 3,213,471
41	0.00077819659	99.9951363557%	$ 3,352,083
42	0.00076290870	99.9952319005%	$ 3,356,070
43	0.00071465591	99.9955334670%	$ 3,377,247
44	0.00047168740	99.9970519801%	$ 3,529,117
45	0.00023958759	99.9985025857%	$ 3,693,291
46	0.00021636486	99.9986477258%	$ 3,712,130
47	0.00012730330	99.9992043567%	$ 3,850,742
48	0.00012446146	99.9992221180%	$ 3,856,323
49	0.00010806509	99.9993245947%	$ 3,889,957
50	0.00006616541	99.9995864667%	$ 4,041,827
51	0.00003242475	99.9997973455%	$ 4,206,001
52	0.00003199147	99.9998000534%	$ 4,209,988
53	0.00001872392	99.9998829755%	$ 4,348,600
54	0.00001688633	99.9998944605%	$ 4,369,777
55	0.00001035884	99.9999352572%	$ 4,521,647
56	0.00000962622	99.9999398361%	$ 4,540,486
57	0.00000941965	99.9999411272%	$ 4,546,067
58	0.00000822879	99.9999485701%	$ 4,579,701
59	0.00000385256	99.9999759215%	$ 4,743,875
60	0.00000206809	99.9999870744%	$ 4,882,487
61	0.00000114105	99.9999928685%	$ 5,034,357
62	0.00000062882	99.9999960699%	$ 5,198,531
63	0.00000057039	99.9999964350%	$ 5,219,708
64	0.00000056015	99.9999964991%	$ 5,223,695
65	0.00000054678	99.9999965826%	$ 5,229,276
66	0.00000047093	99.9999970567%	$ 5,262,910
67	0.00000025223	99.9999984236%	$ 5,401,522
68	0.00000023294	99.9999985441%	$ 5,420,361
69	0.00000011199	99.9999993000%	$ 5,572,231
70	0.00000005738	99.9999996414%	$ 5,736,405
71	0.00000003277	99.9999997952%	$ 5,875,017
72	0.00000003199	99.9999998001%	$ 5,880,598
73	0.00000002769	99.9999998270%	$ 5,914,232
74	0.00000001310	99.9999999181%	$ 6,066,102

Inventory Level (S)	EBO(S)	A(S)	Investment Cost
75	0.00000001150	99.9999999281%	$ 6,087,279
76	0.00000001129	99.9999999294%	$ 6,091,266
77	0.00000000595	99.9999999628%	$ 6,255,440
78	0.00000000551	99.9999999655%	$ 6,274,279
79	0.00000000295	99.9999999816%	$ 6,412,891
80	0.00000000132	99.9999999918%	$ 6,564,761
81	0.00000000127	99.9999999920%	$ 6,570,342
82	0.00000000105	99.9999999934%	$ 6,603,976
83	0.00000000057	99.9999999964%	$ 6,768,150
84	0.00000000053	99.9999999967%	$ 6,789,327
85	0.00000000028	99.9999999982%	$ 6,927,939
86	0.00000000011	99.9999999993%	$ 7,079,809

EBO(S) and A(S) are shown with several decimal places to clearly demonstrate that the values are distinct. That is, marginal difference exists when an additional unit is added to the inventory, even though when the change is, in practice, negligible.

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LIST OF REFERENCES

Albright, S. C. (1989). An approximation to the stationary distribution of a multiechelon repairable-item inventory system with finite sources and repair channels. *Naval Research Logistics*, *36*, 179–195. doi: http://dx.doi.org/10.1002/1520-6750(198904)36:2<179::AID-NAV3220360206>3.0.CO;2-D

Department of Defense. (1991, Mar.28). *DOD Requirements for a logistic support analysis record (MIL-STD-1388-2B)*. Washington, DC: Author.

Diaz, A., & Fu, M. C. (1997). Models for multi-echelon repairable item inventory systems with limited repair capacity. *European Journal of Operational Research*, *97*(3), 480–492. doi: https://doi.org/10.1016/S0377-2217(96)00279-2

Doerr, K. H., & Kang, K. (2014). Bi-criteria risk analysis of domain-specific and cross-domain changes in complex systems. *Computers & Industrial Engineering*, *73*, 51–60. Retrieved from: http://calhoun.nps.edu/bitstream/handle/10945/43121/Doerr_Bi-criteria.pdf?sequence=3

Eruguz, A. S., Tan, T., & Houtum, van, G. J. J. A. N. (2015). A survey of maintenance and service logistics management: classification and research agenda from a maritime sector perspective. *BETA Publicatie: Working Papers* Vol. 489. Eindhoven: Technische Universiteit Eindhoven. Retrieved from: https://pure.tue.nl/ws/files/3815115/24194911834760.pdf

Estado-Maior da Armada (EMA). (2002). EMA-420: *Normas para logística de material* (Rev.2). Brasília, DF: Author.

Estado-Maior da Armada (EMA). (2003). EMA-400: *Manual de logística da marinha* (Rev. 3). Brasília, DF: Author.

Feeney, G. J., & Sherbrooke, C. C. (1966). The (s− 1, s) inventory policy under compound Poisson demand. *Management Science, 12*(5), 391–411. doi: http://dx.doi.org/10.1287/mnsc.12.5.391

Focus Economics. (2017, March 15). Economic snapshot for Latin America. Retrieved from http://www.focus-economics.com/regions/latin-america

Graves, S. C. (1985). A multi-echelon inventory model for a repairable item with one-for-one replenishment. *Management Science*, *31*(10), 1247–1256. doi: http://dx.doi.org/10.1287/mnsc.31.10.1247

Gross, D. (1982). On the ample service assumption of Palm's Theorem in inventory modeling. *Management Science 28*(9):1065–1079. doi: http://dx.doi.org/10.1287/mnsc.28.9.1065

Harris, F. W. (1913). How many parts to make at once. *Operations Research – The Magazine of Management, 38*(6), 947–950. doi: http://dx.doi.org/10.1287/opre.38.6.947

Hillestad, R. J. (1982). *Dyna-METRIC: Dynamic multi-echelon technique for recoverable item control* (R-2785-AF). Santa Monica, CA: RAND Corporation. Retrieved from: http://www.dtic.mil/cgi-bin/GetTRDoc?Location=U2&doc=GetTRDoc.pdf&AD=ADA120446

Hillestad, R. J., & Carrillo, M. J. (1980). *Models and techniques for recoverable item stockage when demand and the repair process are nonstationary - Part I: Performance measurement* (N-1482-AF). Santa Monica, CA: RAND Corporation. Retrieved from: http://www.dtic.mil/cgi-bin/GetTRDoc?Location=U2&doc=GetTRDoc.pdf&AD=ADA091935

Isaacson, K. E., & Boren, P. (1988). *Dyna-METRIC version 5: A capability assessment model including constrained repair and management adaptations* (R-3612-AF). Santa Monica, CA: RAND Corporation. Retrieved from: http://www.dtic.mil/cgi-bin/GetTRDoc?Location=U2&doc=GetTRDoc.pdf&AD=ADA214351

Isaacson, K. E., & Boren, P. M. (1993). *Dyna-METRIC version 6 – An advanced capability assessment model* (R-4214-AF). Santa Monica, CA: RAND Corporation. Retrieved from: https://www.rand.org/content/dam/rand/pubs/reports/2007/R4214.pdf

Jones, J. V. (2006). *Integrated logistics support handbook* (3rd ed.). New York, NY: Sole Logistics Press/McGraw-Hill.

Kang, K., Doerr, K. H., & Sanchez, S. M. (2006, December). A design of experiments approach to readiness risk analysis. In *Proceedings of the 38th Conference on Sinter Simulation* (pp. 1332–1339). Retrieved from: https://www.researchgate.net/publication/221528380

Kececioglu, D. (2002). *Reliability engineering handbook* (Vol. 1). Lancaster, PA: DEStech Publications, Inc.

Lau, H. C., & Song, H. (2008). Multi-echelon repairable item inventory system with limited repair capacity under nonstationary demands. *International Journal of Inventory Research*, *1*(1), 67–92. doi: http://dx.doi.org/10.1504/IJIR.2008.019209

Muckstadt, J. A. (1973). A model for a multi-item, multi-echelon, multi-indenture inventory system. *Management Science, 20*(4-part-i), 472–481. http://dx.doi.org/10.1287/mnsc.20.4.472

Muckstadt, J. A. (2004). *Analysis and algorithms for service parts supply chains.* New York, NY: Springer Science & Business Media.

O'Malley, T. J. (1983). *The aircraft availability model: Conceptual framework and mathematics* (LMI-AF201). Washington, DC: Logistics Management Institute. Retrieved from: http://www.dtic.mil/dtic/tr/fulltext/u2/a132927.pdf

Office of the Assistant Deputy Under Secretary of Defense for Supply Chain Integration (OADUSD[SCI]). (2008, August 08). *Readiness-based sparing roadmap version 1*. Washington, DC: Department of Defense. Retrieved from: http://www.acq.osd.mil/log/sci/rbs/DL739C1-V1_report_final_10Oct08.pdf

Palm, C. (1938). Analysis of the Erlang traffic formula for busy-signal arrangements. *Ericsson Technics, 5*(9), 39–58.

Pyles, R. A. (1984). *The Dyna-METRIC readiness assessment model: motivation, capabilities, and use* (No. RAND/R-2886-AF). Santa Monica, CA: RAND Corporation. Retrieved from: http://www.dtic.mil/cgi-bin/GetTRDoc?Location=U2&doc=GetTRDoc.pdf&AD=ADA145699

Rustenburg, W. D., van Houtum, G. J., & Zijm, W. H. M. (2001). Spare parts management at complex technology-based organizations: An agenda for research. *International Journal of Production Economics, 71*(1), 177–193. doi: http://dx.doi.org/10.1016/S0925-5273(00)00117-1

Secretaria-Geral da Marinha (SGM). (2009). SGM-201: *Normas para o abastecimento na Marinha* (6ª rev.). Brasília, DF: Author.

Sherbrooke, C. C. (1966). Generalization of a queueing theorem of Palm to finite populations. *Management Science*, *12*(11), 907–908. doi: http://dx.doi.org/10.1287/mnsc.12.11.907

Sherbrooke, C. C. (1968). METRIC: A multi-echelon technique for recoverable item control. *Operations Research, 16*(1), 122–141. doi: http://dx.doi.org/10.1287/opre.16.1.122

Sherbrooke, C. C. (1971). An evaluator for the number of operationally ready aircraft in a multilevel supply system. *Operations Research*, *19*(3), 618–635. doi: http://dx.doi.org/10.1287/opre.19.3.618

Sherbrooke, C. C. (1986). VARI-METRIC: Improved approximations for multi-indenture, multi-echelon availability models. *Operations Research*, *34*(2), 311–319. doi: http://dx.doi.org/10.1287/opre.34.2.311

Sherbrooke, C. C. (2004). *Optimal inventory modeling of systems: multi-echelon techniques* (2nd ed.). Boston, MA: Kluwer Academic.

Slay, F. M. (1984). VARI-METRIC: *An approach to modelling multi-echelon resupply when the demand process is Poisson with a gamma prior* (AF301-3). Washington, DC: Logistic Management Institute.

Slay, F. M., & King, R. M. (1987). *Prototype aircraft sustainability model* (AF-601-R2). Bethesda, MD: Logistics Management Institute. Retrieved from: http://www.dtic.mil/cgi-bin/GetTRDoc?Location=U2&doc=GetTRDoc.pdf&AD=ADA183183

Sleptchenko, A., Van der Heijden, M. C., & Van Harten, A. (2002). Effects of finite repair capacity in multi-echelon, multi-indenture service part supply systems. *International Journal of Production Economics, 79*(3), 209–230. doi: http://doi.org/10.1016/S0925-5273(02)00155-X

Under Secretary of Defense for Acquisition, Technology, and Logistics (USD(AT&L)). (2014). *DOD Supply chain materiel management procedures: demand and supply planning.* Washington, DC: Department of Defense. Accessed from: http://www.dtic.mil/whs/directives/corres/pdf/414001m/414001m_vol02.pdf

van Houtum, G. J., & Kranenburg, B. (2015). *Spare parts inventory control under system availability constraints* (Vol. 227). New York, NY: Springer Science & Business Media.

Weifenbach, A. (1966). *Base maintenance activity and repair cycle times* (RM-5027-PR). Santa Monica, CA: RAND Corporation.

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